

Lesson 5: SLR-ish: Categorical Covariates

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2025-01-22

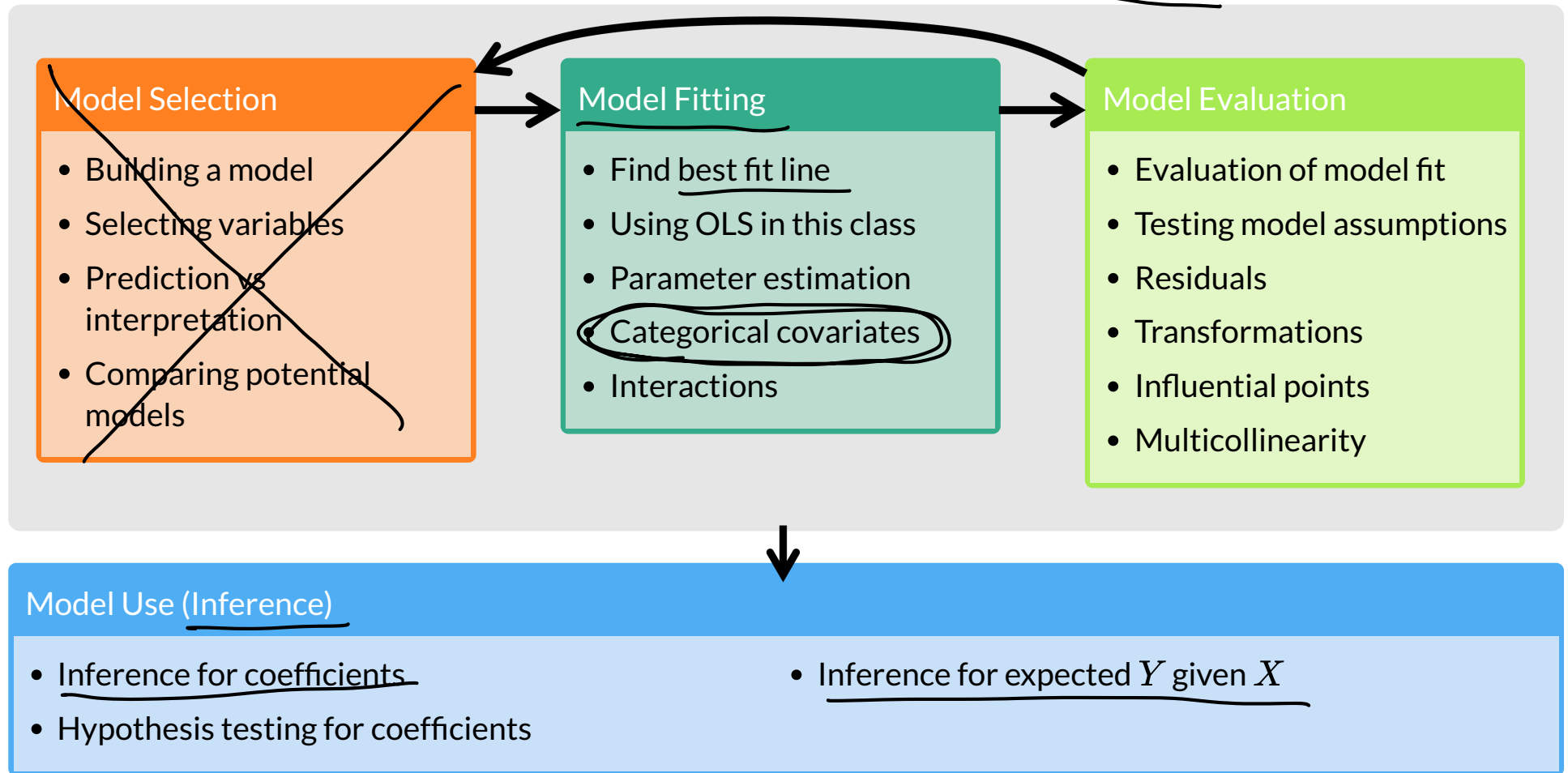
Learning Objectives

1. Understand why we need a new way to code categorical variables compared to continuous variables
2. Write the regression equation for a categorical variable using reference cell coding
3. Calculate and interpret coefficients for reference cell coding
4. Change the reference level in a categorical variable for reference cell coding
5. Create new variables and interpret coefficient for ordinal / scoring coding

Why “SLR-ish”?

- The **strict** definition of simple linear regression: only two variables that are BOTH continuous
- Common (but kinda wrong) use of simple linear regression: only two variables with outcome continuous and predictor not specified
- I’m including multi-level categorical covariates in SLR mostly because it’s easier to learn now!

Let's map that to our regression analysis process SLR



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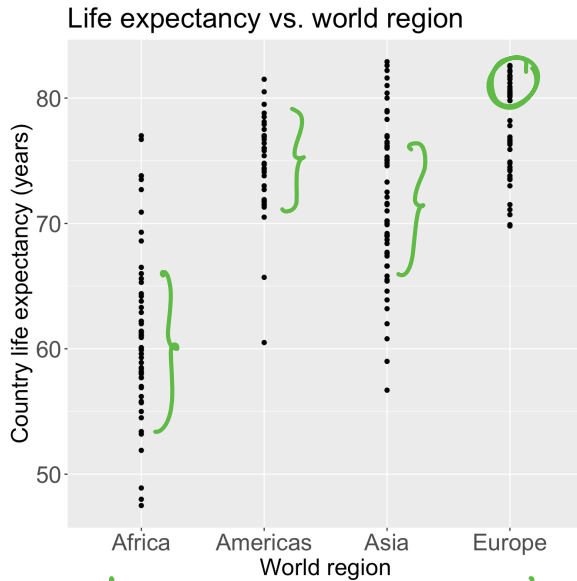
Still looking at Gapminder Life Expectancy data

- We will look at life expectancy vs. these world regions
- Gapminder uses four world regions
 - Africa
 - The Americas
 - Asia
 - Europe
- World region is a multi-level categorical covariate: it has four regions
more than 2
- Note: I am calling the expected life expectancy $\widehat{LE}$ *LE*
 - Previously, I have written $\widehat{\text{life expectancy}}$

Linear regression with a categorical covariate (1/2)

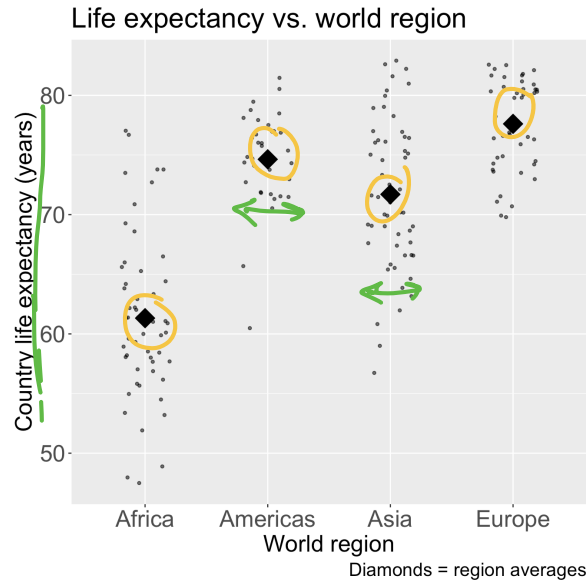
Bad option for visualization:

► Code



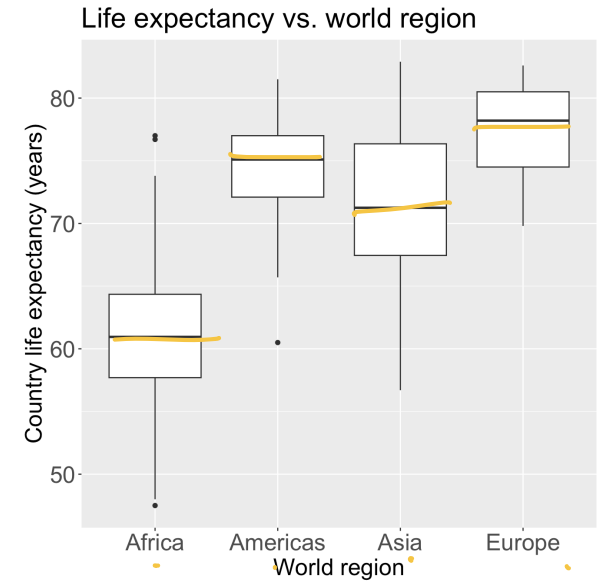
Good option for visualization:

► Code



Good option for visualization:

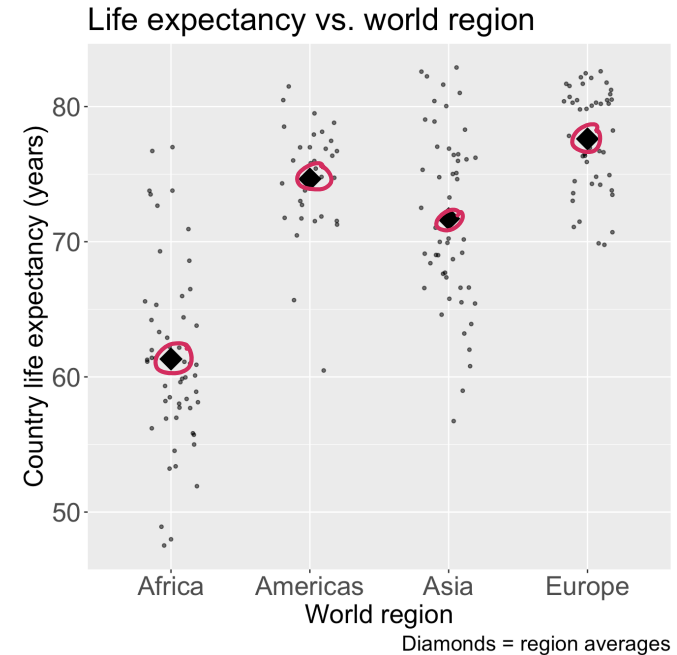
► Code



- Used `geom_jitter()`

Linear regression with a categorical covariate (2/2)

- When using a categorical covariate/predictor (that is not ordered),
 - We do NOT, technically, find a best-fit line
 - Instead we model the means of the outcome
 - For the different levels of the categorical variable
- NP* *P*
- In 511, we used Kruskal-Wallis test and our ANOVA table to test if groups means were statistically different from one another
 - We can do this **using linear models** AND we can include other variables in the model



There are different ways to code categorical variables

- ★ • Reference cell coding (sometimes called dummy coding)
 - Compares each level of a variable to the omitted (reference) level
- Effect coding (sometimes called sum coding or deviation coding)
 - Compares deviations from the grand mean
 - Not covered in our class
- Ordinal encoding (sometimes called scoring)
 - Categories have a natural, even spaced ordering

If you want to learn more about these and other coding schemes:

- Coding Systems for Categorical Variables in Regression Analysis
- Categorical Data Encoding Techniques
- Coding Schemes for Categorical Variables

Building the regression equation: problem with a single coefficient

Previously: simple linear regression

- Outcome Y = numerical variable
- Predictor X = numerical variable

The regression (best-fit) line is:

$$\hat{Y} = \hat{\beta}_0 + \hat{\beta}_1 \cdot X$$

New: what if the explanatory variable is categorical?

Naively, we could write: $\hat{Y} = \hat{\beta}_0 + \hat{\beta}_1 \cdot X$

Or, with our variables:

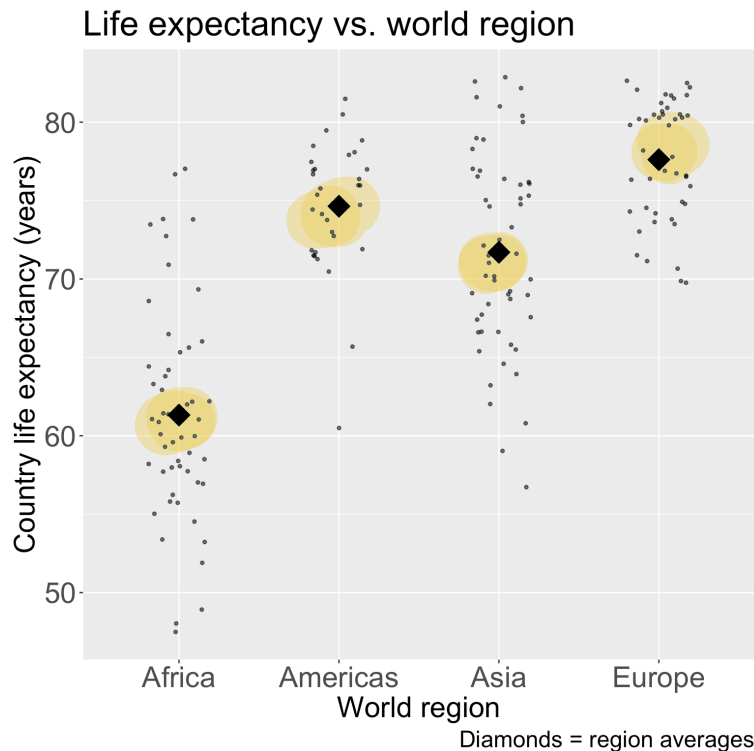
$\text{X} \rightarrow [\widehat{\text{LE}} = \hat{\beta}_0 + \hat{\beta}_1 \cdot \text{WR}]$

- But what does **WR** (world regions) mean in this equation?
 - What values can it take? How do we represent each region?

• Note: the above is **WRONG**

Building the regression equation: how do we map categories to means?

- If we only have world region in our model and want to map it to an expected life expectancy...
- We want to create a function that can map each region to life expectancy
 - If in Africa: $\widehat{LE} = 61.32$ years
 - If in the Americas: $\widehat{LE} = 74.64$ years
 - If in Asia: $\widehat{LE} = 71.70$ years
 - If in Europe: $\widehat{LE} = 77.61$ years
- Can we make one equation for $\widehat{LE}$ by putting the “if” statements within the equation?



Building the regression equation: Indicator functions

- In order to represent each region in the equation, we need to introduce a new function:

- Indicator function:

if $X = x$

$$I(X = x) \text{ or } I(x) = \begin{cases} 1, & \text{if } X = x \\ 0, & \text{else } X \neq x \end{cases}$$

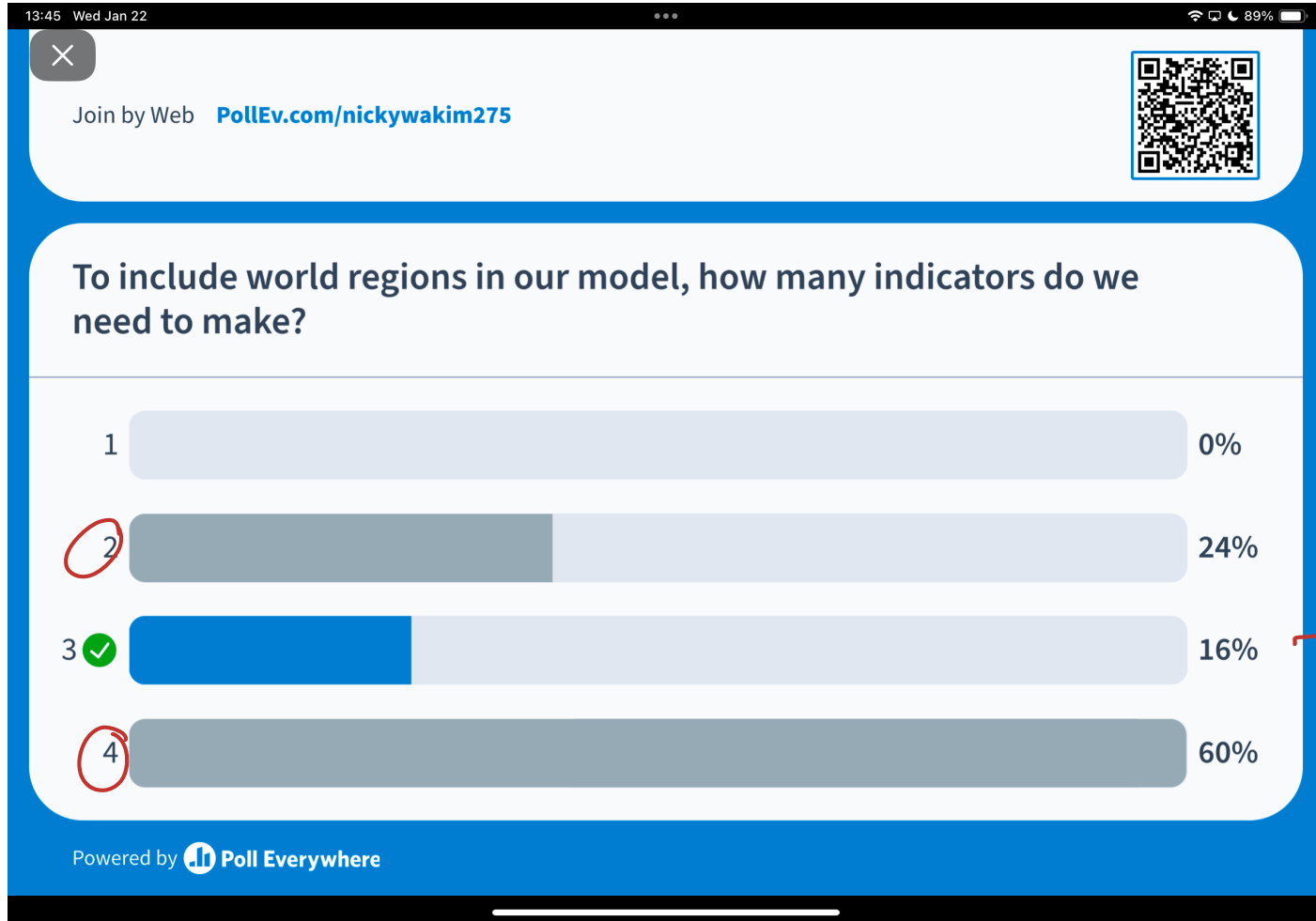
- This basically a binary yes/no if X is a specific value x
- For example, if we want to identify a country ~~being~~ in the Americas region, we can make:

if $WR = \text{Americas}$ is $WR = \text{Americas}$ or $I(\text{Americas}) = \begin{cases} 1, & \text{if } WR = \text{Americas} \\ 0, & \text{else} \end{cases}$

	city	Region
country	1	<u>Americas</u>
	2	<u>Asia</u>
	3	<u>Africa</u>

$I(WR = \text{Americas})$	$I(\text{Asia})$
1	0
0	1
0	0

Poll Everywhere Question 1



one becomes
our reference

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Building the regression equation: Indicators in our equation

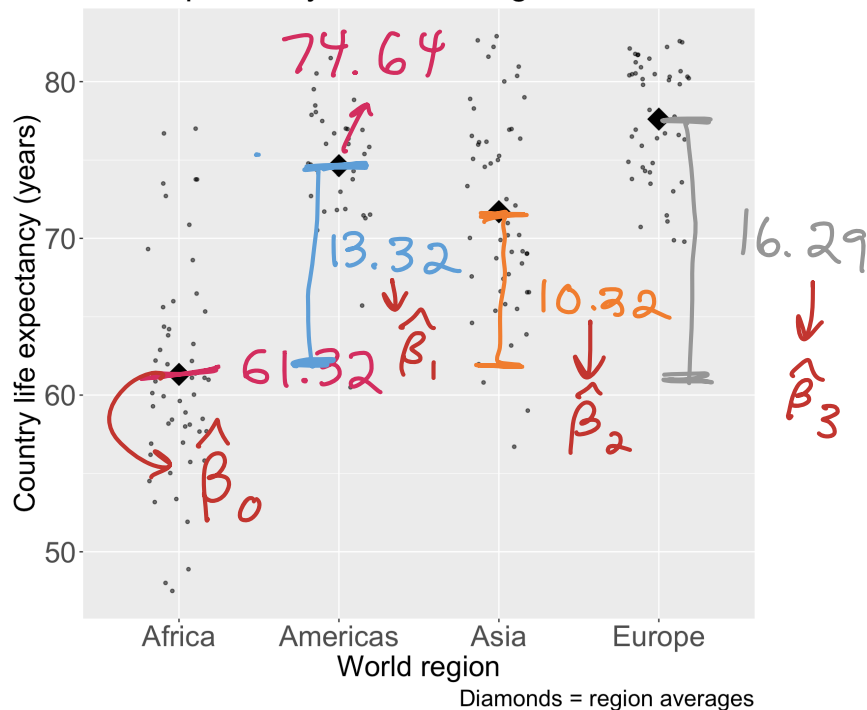
$$\widehat{LE} = 61.32 \cdot I(\text{Africa}) + 74.64 \cdot I(\text{Americas}) + 71.7 \cdot I(\text{Asia}) + 77.61 \cdot I(\text{Europe})$$

- However, a linear regression equation still requires an intercept!
 - So one of our regions need to become our “reference” group
 - We’ll use Africa as our reference
 - That means we need to adjust all the numbers

$$\widehat{LE} = 61.32 + 13.32 \cdot I(\text{Americas}) + 10.38 \cdot I(\text{Asia}) + 16.29 \cdot I(\text{Europe})$$

$$\widehat{LE} = \hat{\beta}_0 + \hat{\beta}_1 \cdot I(\text{Americas}) + \hat{\beta}_2 \cdot I(\text{Asia}) + \hat{\beta}_3 \cdot I(\text{Europe})$$

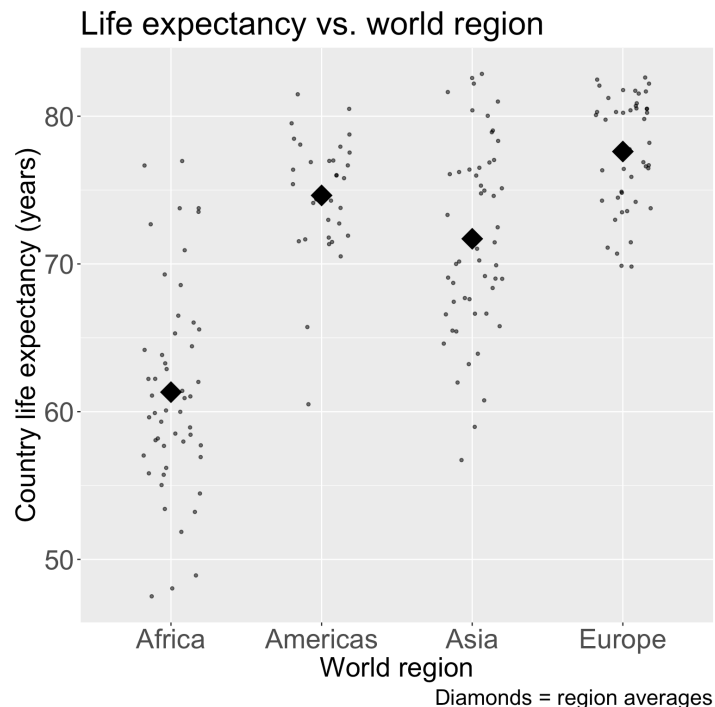
Life expectancy vs. world region



Viewing the regression equation another way

$$\widehat{LE} = \widehat{\beta}_0 + \widehat{\beta}_1 \cdot I(\text{Americas}) + \widehat{\beta}_2 \cdot I(\text{Asia}) + \widehat{\beta}_3 \cdot I(\text{Europe})$$

<u>World region</u>	<u>Regression equation for WR</u>	Average Life Expectancy for WR
Africa	$\widehat{LE} = \widehat{\beta}_0 + \widehat{\beta}_1 \cdot 0 + \widehat{\beta}_2 \cdot 0 + \widehat{\beta}_3 \cdot 0$	$\widehat{LE} = \widehat{\beta}_0$
Americas	$\widehat{LE} = \widehat{\beta}_0 + \widehat{\beta}_1 \cdot 1 + \widehat{\beta}_2 \cdot 0 + \widehat{\beta}_3 \cdot 0$	$\widehat{LE} = \widehat{\beta}_0 + \widehat{\beta}_1$
Asia	$\widehat{LE} = \widehat{\beta}_0 + \widehat{\beta}_1 \cdot 0 + \widehat{\beta}_2 \cdot 1 + \widehat{\beta}_3 \cdot 0$	$\widehat{LE} = \widehat{\beta}_0 + \widehat{\beta}_2$
Europe	$\widehat{LE} = \widehat{\beta}_0 + \widehat{\beta}_1 \cdot 0 + \widehat{\beta}_2 \cdot 0 + \widehat{\beta}_3 \cdot 1$	$\widehat{LE} = \widehat{\beta}_0 + \widehat{\beta}_3$



Learning Objectives

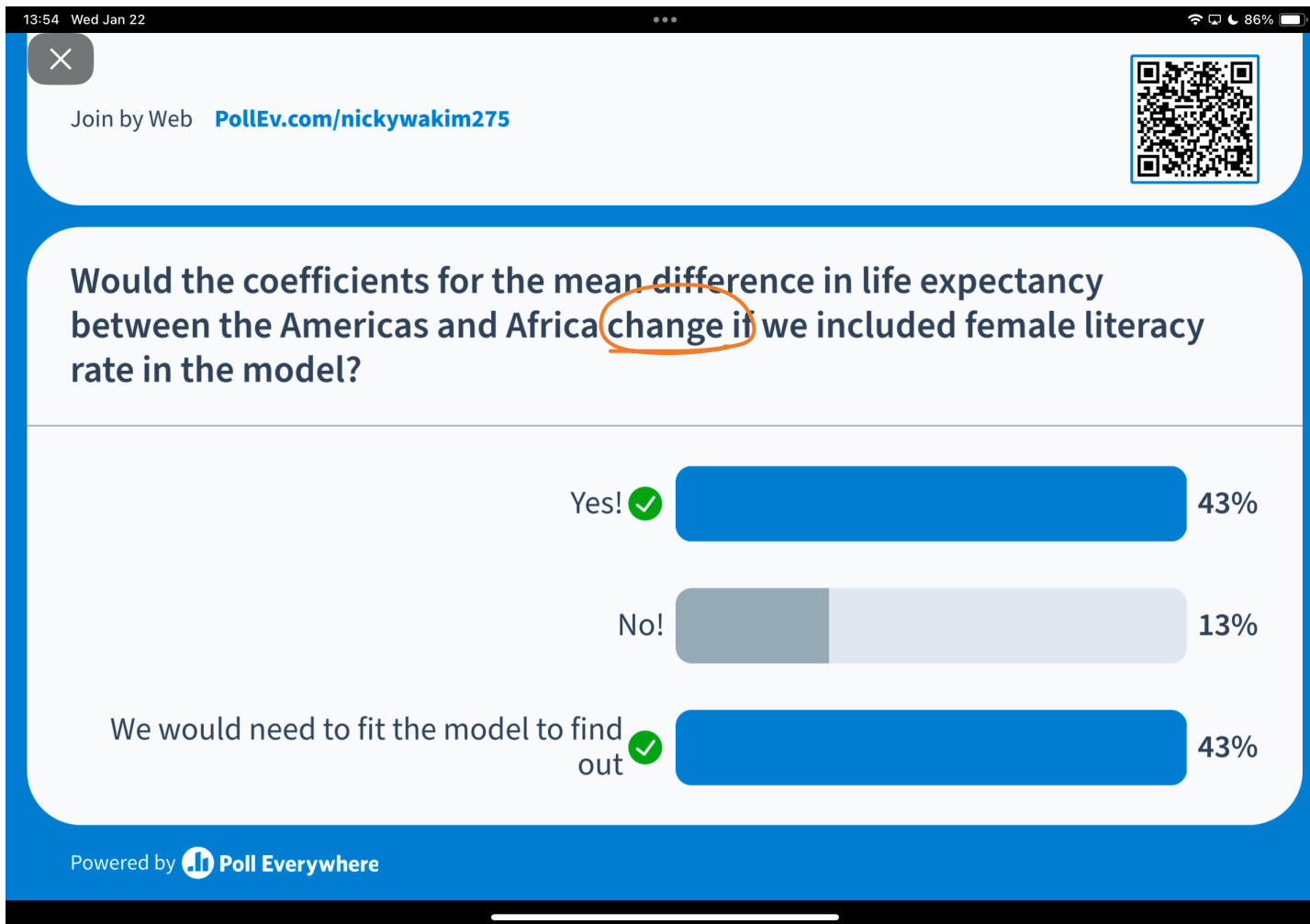
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Interpretation of regression equation coefficients

- Remember: expected, mean, and average are interchangeable

Coefficient	Interpretation
$\hat{\beta}_0$	Expected ^{or} /mean ^{or} /average life expectancy of Africa
$\hat{\beta}_1$	<u>Difference in mean life expectancy of the Americas and Africa</u> -OR- <u>Mean difference in life expectancy of the Americas and Africa</u>
$\hat{\beta}_2$	<u>Difference in mean life expectancy between Asia and Africa</u> -OR- <u>Mean difference in life expectancy between Asia and Africa</u>
$\hat{\beta}_3$	<u>Difference in mean life expectancy between Europe and Africa</u> -OR- <u>Mean difference in life expectancy between Europe and Africa</u>

Poll Everywhere Question 2



$$\hat{LE} = \hat{\beta}_0 + \hat{\beta}_1 FLR$$

$$\hookrightarrow FLR = 0$$

$$\hat{LE} = \hat{\beta}_0 + \hat{\beta}_1 I(Am) + \hat{\beta}_2 I(As) + \hat{\beta}_3 I(E)$$

$\hat{LE}$ country in Africa

Regression table with `lm()` function

```
1 mod11 <- lm(LifeExpectancyYrs ~ four_regions, data = gapm2)
2 tidy(mod11, conf.int=T) %>% gt() %>% tab_options(table.font.size = 38) %>%
3   fmt_number(decimals = 2)
```

term	estimate	std.error	statistic	p.value	conf.low	conf.high
(Intercept)	61.32	0.76	80.26	0.00	59.81	62.83
four_regionsAmericas	13.32	1.23	10.83	0.00	10.89	15.74
four_regionsAsia	10.38	1.08	9.61	0.00	8.25	12.51
four_regionsEurope	16.29	1.13	14.37	0.00	14.05	18.52

$$\widehat{LE} = 61.32 + 13.32 \cdot I(\text{Americas}) + 10.38 \cdot I(\text{Asia}) + 16.29 \cdot I(\text{Europe})$$

- Which world region did R choose as the reference level?
- How you would calculate the mean life expectancies of world regions using *only* the results from the regression table?

Bringing in the numbers/units/95% CI

Coefficient	Interpretation
$\hat{\beta}_0$	Average life expectancy of countries in Africa is 61.32 years (95% CI: 59.81, 62.83).
$\hat{\beta}_1$	The difference in mean life expectancy between countries in the Americas and Africa is 13.32 (95% CI: 10.89, 15.74).
$\hat{\beta}_2$	The difference in mean life expectancy between countries in the Americas and Africa is 10.38 (95% CI: 8.25, 12.51).
$\hat{\beta}_3$	The difference in mean life expectancy between countries in Europe and Africa is 16.29 (95% CI: 14.05, 18.52).

- Don't forget that we can use the confidence intervals to assess whether the mean difference with Africa is significant or not

We can also use R to report each region's average life expectancy

Find the 95% CI's for the mean life expectancy for the Americas, Asia, and Europe

- Use the base R `predict()` function (see Lesson 4 for more info)
- Requires specification of a `newdata` "value"

```
1 newdata <- data.frame(four_regions = c("Africa", "Americas", "Asia", "Europe"))
1 (pred = predict(model1,
2               newdata=newdata,
3               interval="confidence"))
```

	fit	lwr	upr	
1	61.32037	59.81287	62.82787	Africa
2	74.63824	72.73841	76.53806	Americas
3	71.70185	70.19435	73.20935	Asia
4	77.60889	75.95751	79.26027	- Europe

Interpretations

- The average life expectancy for countries in the Americas is 74.64 years (95% CI: 72.74, 76.54).
- The average life expectancy for countries in Asia is 71.7 years (95% CI: 70.19, 73.21).
- The average life expectancy for countries in Europe is 77.61 years (95% CI: 75.96, 79.26).

Another way to look at coefficient values

$$\widehat{\text{LE}} = \hat{\beta}_0 + \hat{\beta}_1 \cdot I(\text{Americas}) + \hat{\beta}_2 \cdot I(\text{Asia}) + \hat{\beta}_3 \cdot I(\text{Europe})$$

► Code

<u>World regions</u>	<u>Average life expectancy</u>	Difference with Africa
Africa	61.3	<u>0.0</u>
Americas	74.6	<u>13.3</u>
Asia	71.7	<u>10.4</u>
Europe	77.6	<u>16.3</u>

$$\widehat{\text{LE}} = 61.32 + 13.32 \cdot I(\text{Americas}) + 10.38 \cdot I(\text{Asia}) + 16.29 \cdot I(\text{Europe})$$

10 minute break here?

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Reference levels

Why is **Africa** not one of the variables in the regression equation?

$$\widehat{\text{LE}} = \hat{\beta}_0 + \hat{\beta}_1 \cdot I(\text{Americas}) + \hat{\beta}_2 \cdot I(\text{Asia}) + \hat{\beta}_3 \cdot I(\text{Europe})$$

- Categorical variables have to have at least 2 levels. If they have 2 levels, we call them *binary*
- We choose one level as our reference level to which all other levels of the categorical variable are compared
 - The levels Americas, Asia, Europe are compared to the level Africa
- The **intercept** of the regression equation is the *mean of the outcome restricted to the reference level*
 - Recall that the intercept is the mean life expectancy of Africa, which was our reference level
- If the categorical variable has r levels, then we need $r - 1$ variables/coefficients to model it!

We can change the reference level to Europe (1/2)

- Suppose we want to compare the mean life expectancies of world regions to the Europe level instead of Africa
- Below is the estimated regression equation for when Africa is the reference level

$$\widehat{\text{LE}} = \hat{\beta}_0 + \hat{\beta}_1 \cdot \underline{I(\text{Americas})} + \hat{\beta}_2 \cdot \underline{I(\text{Asia})} + \hat{\beta}_3 \cdot \underline{I(\text{Europe})}$$

- Update the variables to make Europe the reference level:

$$\widehat{\text{LE}} = \hat{\beta}_0 + \hat{\beta}_1 \cdot \underline{I(\text{Africa})} + \hat{\beta}_2 \cdot \underline{I(\text{Americas})} + \hat{\beta}_3 \cdot \underline{I(\text{Asia})}$$

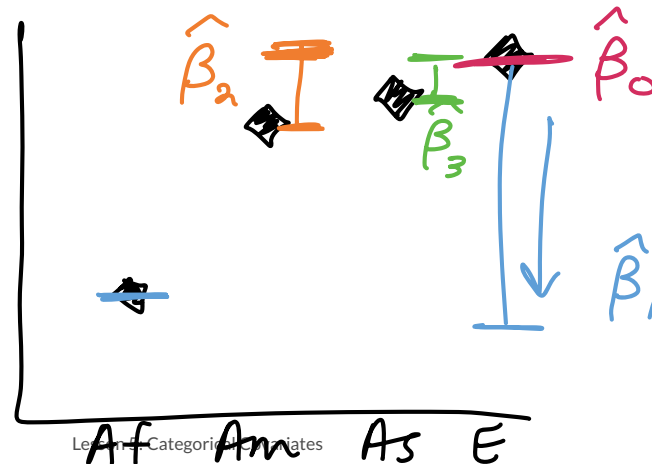
↑
Europe

We can change the reference level to Europe (2/2)

- Now update the coefficients of the regression equation using the output below.

World regions	Average life expectancy	Difference with Europe
Africa	61.32	-16.29
Americas	74.64	-2.97
Asia	71.70	-5.91
Europe	77.61	0.00

$$\widehat{LE} = 77.61 - 16.29 \cdot I(\text{Africa}) - 2.97 \cdot I(\text{Americas}) - 5.91 \cdot I(\text{Asia})$$



R: Change reference level to Europe (1/2)

- `four_regions` data type was originally a `character` - check this with `str()`

```
1 str(gapm$four_regions)
```

→ chr [1:195] "asia" "europa" "africa" "europa" "africa" "americas" ...

- In order to change the reference level, we need to convert it to data type `factor`

- I also did this at the beginning to capitalize each region

```
1 gapm_ex = gapm %>%  
2   mutate(  
3     four_regions = factor(four_regions,  
4       levels = c("africa", "americas", "asia", "europa"),  
5       labels = c("Africa", "Americas", "Asia", "Europe"))  
6   )  
7 str(gapm_ex$four_regions)
```

turn this variable into a factor

currently exists

Factor w/ 4 levels "Africa","Americas",...: 3 4 1 4 1 2 2 4 3 4 ...

```
1 levels(gapm_ex$four_regions) # order of factor levels
```

```
[1] "Africa"    "Americas" "Asia"      "Europe"
```

R: Change reference level to Europe (2/2)

- Now change the order of the factor levels
- Code below uses fct_relevel() from the forcats package that gets loaded as a part of the tidyverse
- Any levels not mentioned will be left in their existing order, after the explicitly mentioned levels.

```
1 gapm2 <- gapm2 %>%  
2   mutate(  
3     four_regions = fct_relevel(four_regions, "Europe")  
4   )
```

↳ E, A_s, Am, Af

- Check the order:

```
1 levels(gapm2$four_regions)  
[1] "Europe" "Africa" "Americas" "Asia"
```

R: Run model with Europe as the reference level

```
1 levels(gapm2$four_regions)
[1] "Europe" "Africa" "Americas" "Asia"

1 model2 <- lm(LifeExpectancyYrs ~ four_regions, data = gapm2)
2 tidy(model2) %>% gt() %>% tab_options(table.font.size = 35) %>%
3   fmt_number(decimals = 2)
```

Europe —

term	estimate	std.error	statistic	p.value
(Intercept)	77.61	0.84	92.72	0.00
four_regionsAfrica	-16.29	1.13	-14.37	0.00
four_regionsAmericas	-2.97	1.28	-2.33	0.02
four_regionsAsia	-5.91	1.13	-5.21	0.00

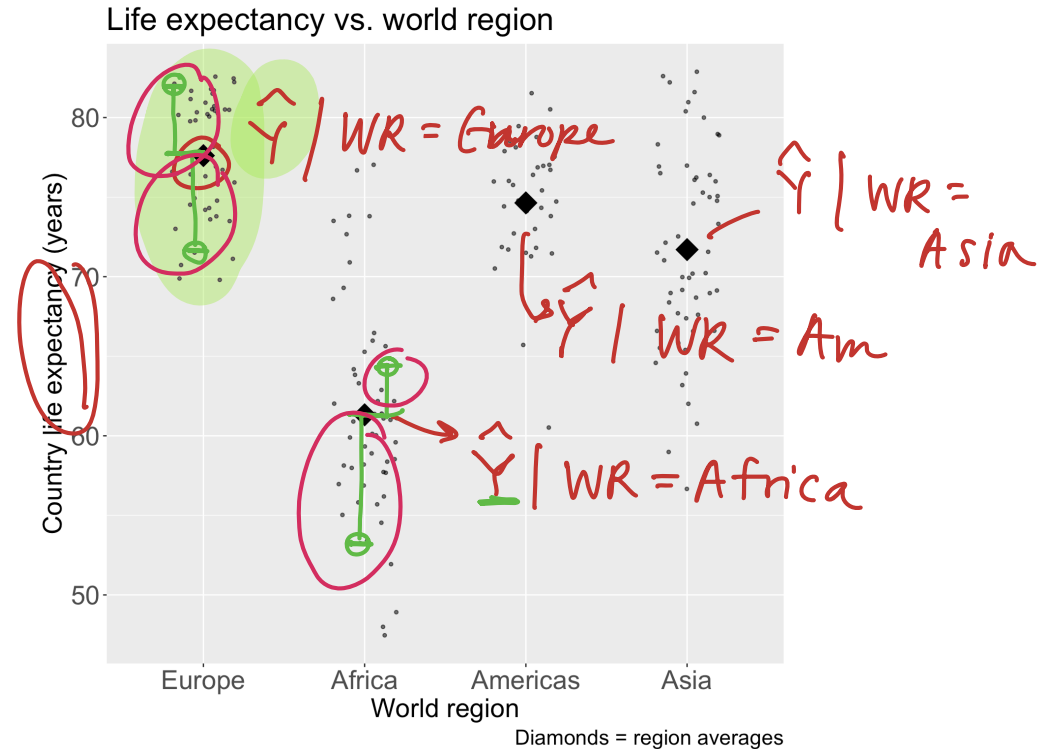
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$$\widehat{LE} = 77.61 - 16.29 \cdot I(\text{Africa}) - 2.97 \cdot I(\text{Americas}) - 5.91 \cdot I(\text{Asia})$$

Fitted values & residuals

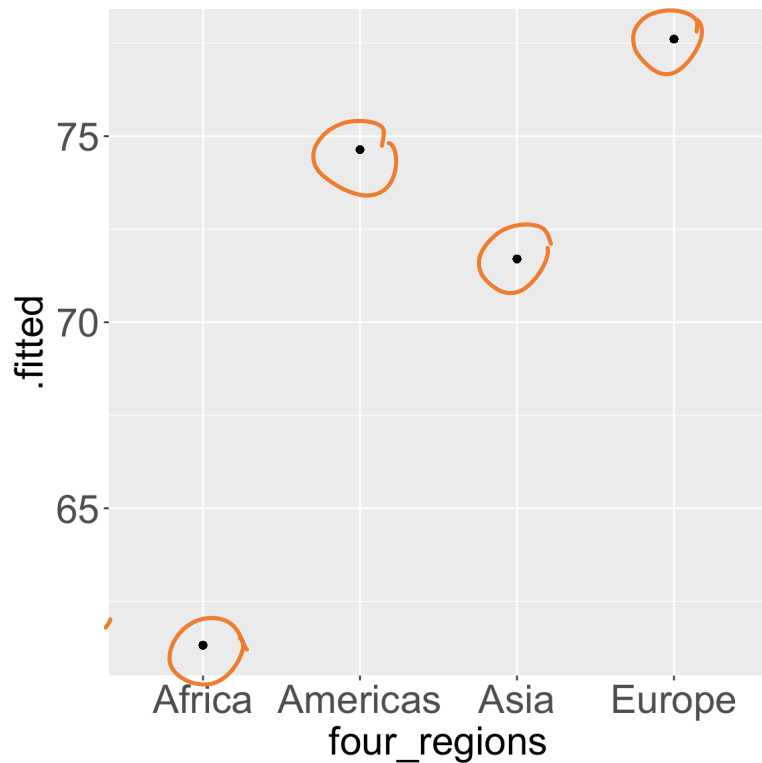
Similar to as before:

- **Observed values** Y are the values in the dataset
- **Fitted values** $\hat{Y}$ are the values that ~~fall on the best fit line for a specific value of x~~ are the means of the outcome stratified by the categorical predictor's levels
- **Residuals** ($\hat{\epsilon} = Y - \hat{Y}$) are the differences between the two



Fitted values are the same as the means

```
1 m1_aug <- augment(model1)
2
3 ggplot(m1_aug, aes(x = four_regions, y = .fitted)) + geom_point() +
4   theme(axis.text = element_text(size = 22), axis.title = element_text(size = 22))
```

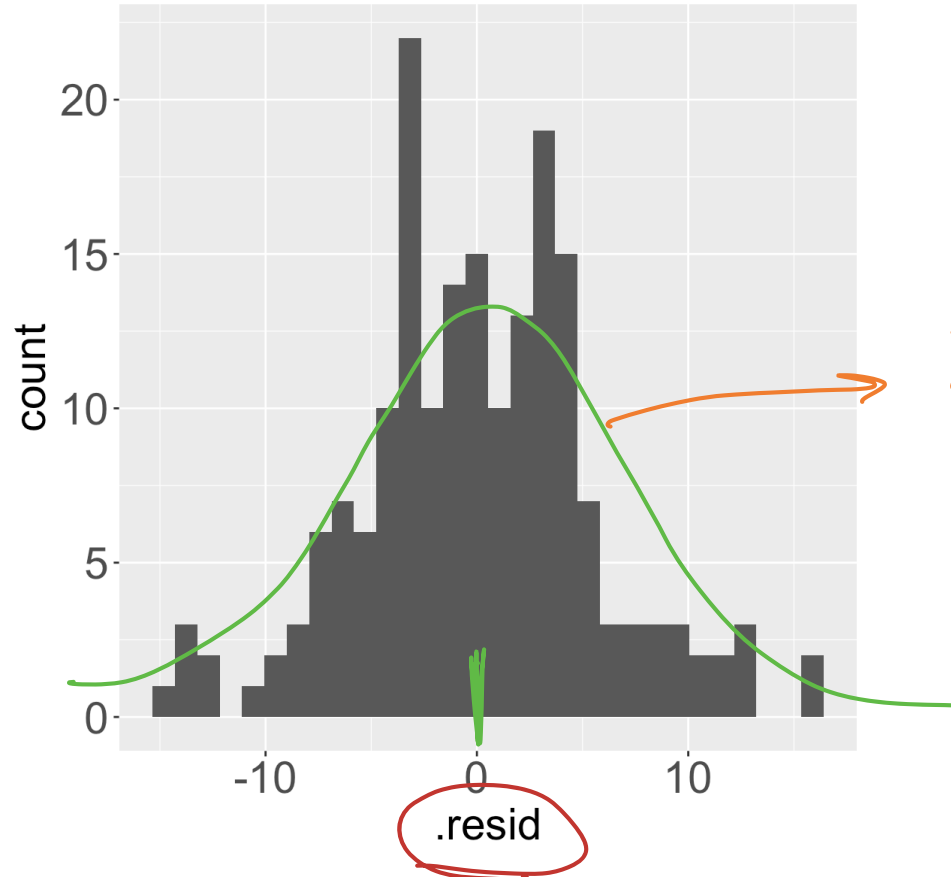


Residual plots (now the spread within each region)

```
1 ggplot(ml_aug, aes(x=.resid)) + geom_histogram() +  
2   theme(axis.text = element_text(size = 22), title = element_text(size = 22))
```

theory

$$\varepsilon \sim N(0, \sigma^2)$$



$$\hat{\varepsilon} \sim N(0, \hat{\sigma}^2)$$

Poll Everywhere Question 3

14:37 Wed Jan 22



Join by Web PollEv.com/nickywakim27

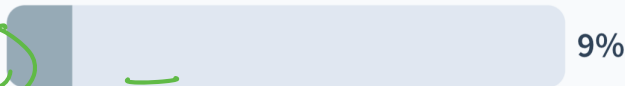
"polleverywhere.com" is in full screen.
Swipe down to exit.



What does the plot of the residuals visualize for our regression model:

$$\widehat{LE} = \hat{\beta}_0 + \hat{\beta}_1 \cdot I(\text{Africa}) + \hat{\beta}_2 \cdot I(\text{Americas}) + \hat{\beta}_3 \cdot I(\text{Asia})$$

The deviation between observed life expectancy and the overall mean life expectancy



The deviation between observed life expectancy and each country's region's mean life expectancy ✓



The deviation between overall mean life expectancy and each region's mean life expectancy



Powered by Poll Everywhere

$$\epsilon_i = \underbrace{Y_i}_{\text{observed country's LE}} - \underbrace{(\hat{Y} | X_i)}_{\substack{\uparrow \\ WR_i \\ \text{mean LE given WR} \\ \text{(of a specific country)}}}$$

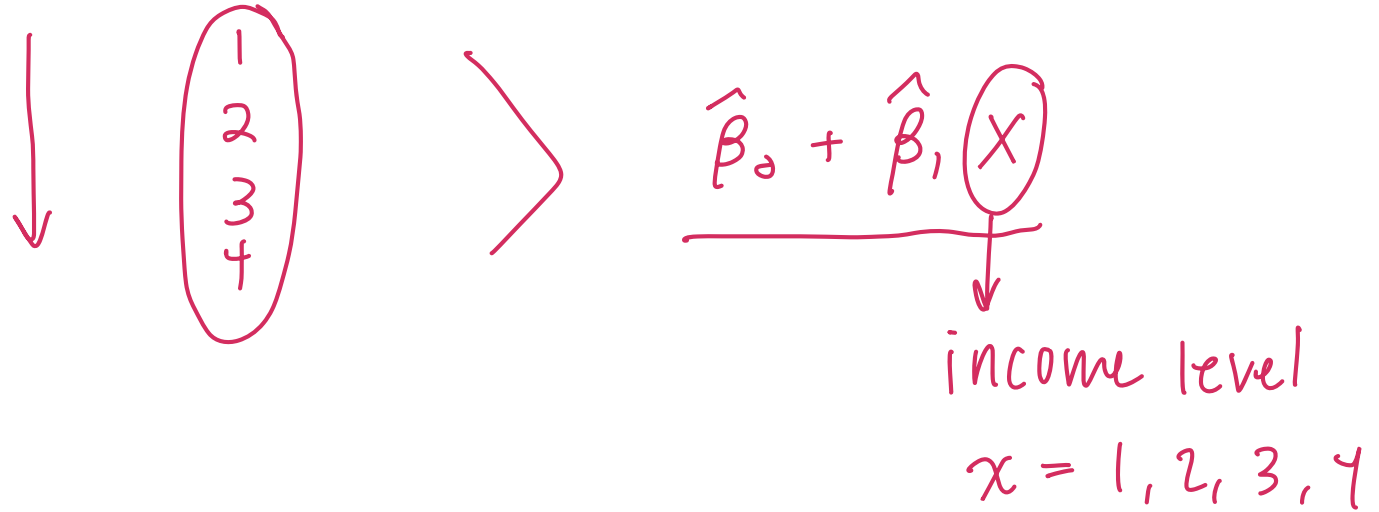
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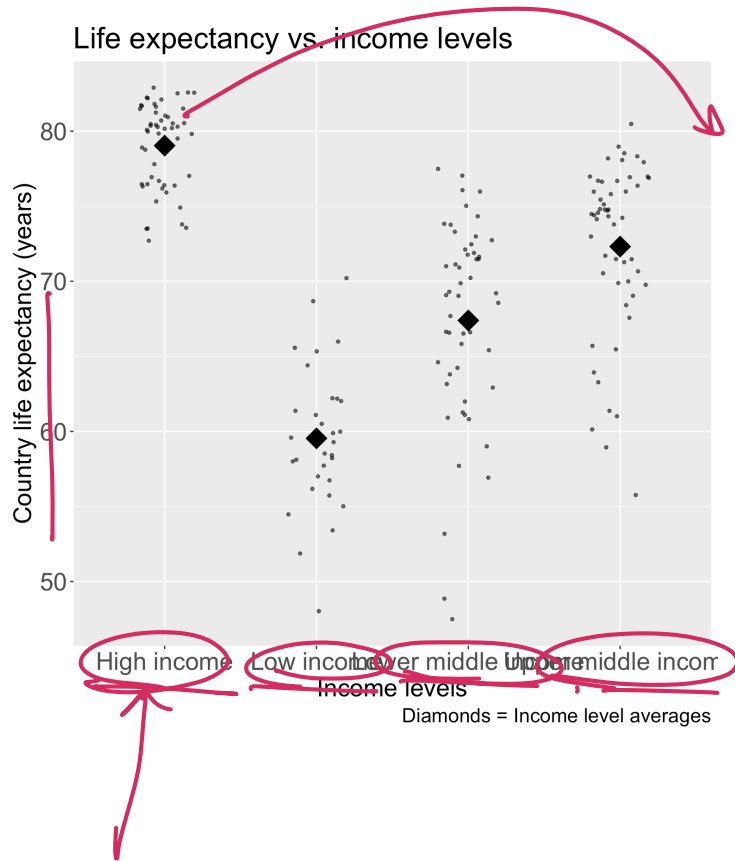
Let's look at life expectancy vs. four income levels

- Gapminder discusses individual income levels
- Income levels for a country** is based on average GDP per capita, and grouped into:

- Low income
- Lower middle income
- Upper middle income
- High income



Visualizing the ordinal variable, income levels



A few changes needed:

- Put the income levels in order

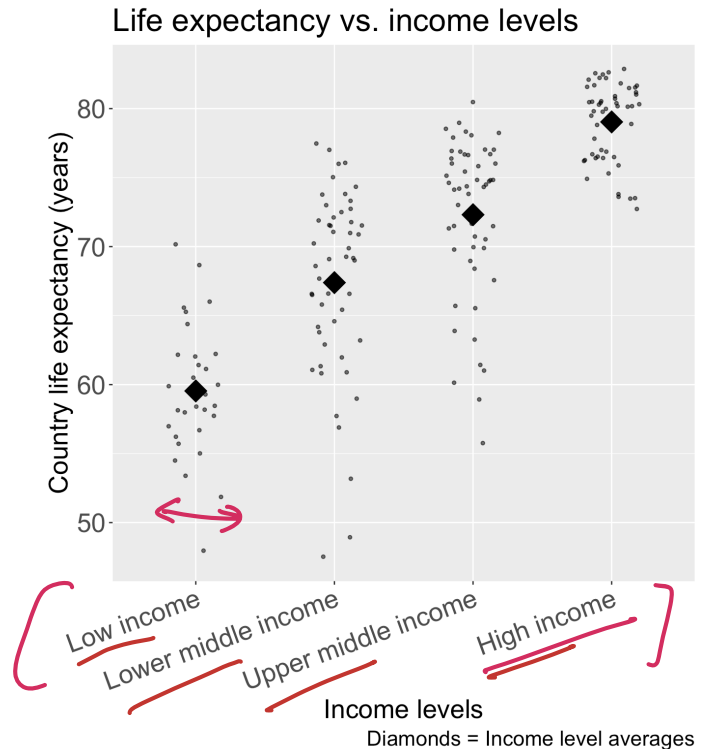
```
1 gapm2 = gapm2 %>%  
2   mutate(income_levels = factor(income_levels,  
3                                   ordered = T,  
4                                   levels = c("Low income",  
5                                               "Lower middle income",  
6                                               "Upper middle income",  
7                                               "High income")))
```

- Make the income levels readable
 - How to Rotate Axis Labels in ggplot2?

Much better: Visualizing the ordinal variable, income levels

*dia
monds*

```
1 ggplot(gapm2, aes(x = income_levels, y = LifeExpectancyYrs)) +  
2 geom_jitter(size = 1, alpha = .6, width = 0.2) +  
3 stat_summary(fun = mean, geom = "point", size = 8, shape = 18) +  
4 labs(x = "Income levels",  
5       y = "Country life expectancy (years)",  
6       title = "Life expectancy vs. income levels",  
7       caption = "Diamonds = Income level averages") +  
8 theme(axis.title = element_text(size = 20),  
9       axis.text = element_text(size = 20),  
10      title = element_text(size = 20),  
11      axis.text.x=element_text(angle = 20, vjust = 1, hjust=1))
```



How can we code this variable?

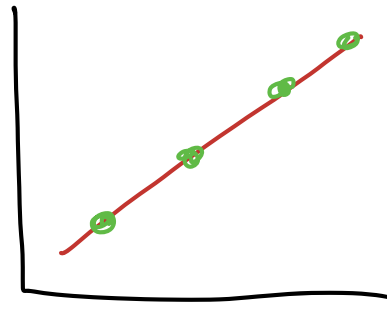
We have two options:

Treat the levels as nominal and use reference cell coding

- Like we did with world regions
- This option will not break the linearity assumption
- For g categories of the variable, we will have $g - 1$ coefficients to estimate

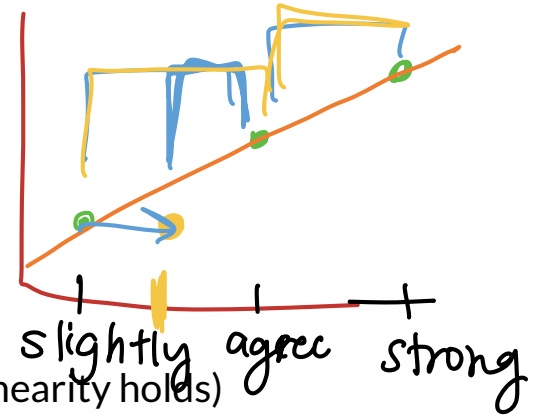
Use the ordinal values to score the levels and treat as a numerical variable

- Even if a variable is inherently ordered, we need to check that linearity holds if categories are represented as numbers
- This way of coding preserves more power in the model (less coefficients to estimate means more power)
- Only one coefficient to estimate



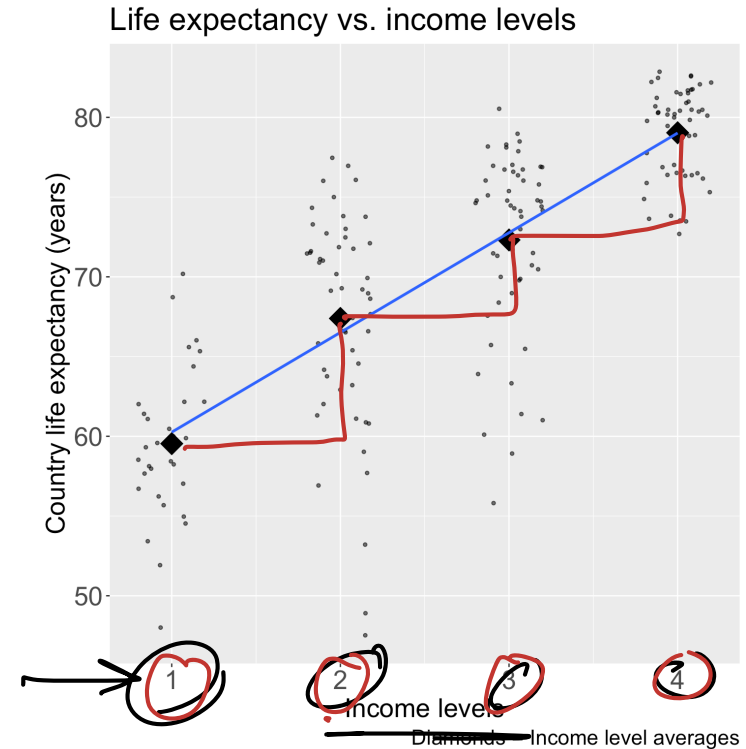
Some important considerations when scoring ordinal variables

- Even if a variable is inherently ordered, we need to check that linearity holds if categories are represented as numbers (more in next lessons)
 - Linearity is an assumption of linear regression: that the relationship between X and Y is linear
- Assumes differences between adjacent groups are equal
 - Income levels are pre-set groups by Gapminder
 - Might be hard to interpret “every 1-level increase in income level”
- Is the variable part of the main relationship that you are investigating? (even if linearity holds)
 - If yes, consider leaving as reference cell coding unless the interpretation makes sense
 - If no, and just needed as an adjustment in your model, then power benefit of scoring might be worth it!



Check that linearity holds for income levels

- Using visual assessment, linearity holds for our income levels (more in next lessons)
- We can use the ordinal encoding for income levels



Poll Everywhere Question 4

14:54 Wed Jan 22

...

67%



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In your IAT score project, for which variables might you consider using an ordinal coding/scoring?

something
scale age category
bmi groups bins **weight** best
rating age-bins self thinness-fatness
reported likert age-groups
worst self-perception

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Ordinal coding / Scoring

- Map each income level to a number
- Usually start at 1

Income Level	Score
Low income	1
Lower middle income	2
Upper middle income	3
High income	4

```
1 gapm2 = gapm2 %>%  
2   mutate(income_num = as.numeric(income_levels))  
3   str(gapm2$income_num)
```

```
num [1:187] 1 3 3 4 2 4 3 2 4 4 ...
```

factor variable
w/ set levels

Run the model with the scored income

```
1 mod_inc2 = lm(LifeExpectancyYrs ~ income_num, data = gapm2)
2 tidy(mod_inc2) %>% gt() %>% tab_options(table.font.size = 37) %>%
3   fmt_number(decimals = 2)
```

	term	estimate	std.error	statistic	p.value
$\hat{\beta}_0$	(Intercept)	54.01	1.06	51.03	0.00
$\hat{\beta}_1$	income_num	6.25	0.37	16.91	0.00

$$\widehat{LE} = \hat{\beta}_0 + \hat{\beta}_1 \cdot \text{Income level}$$
$$\widehat{LE} = 54.01 + 6.25 \cdot \text{Income level}$$

- Keep in mind: We cannot calculate the expected outcome outside of the scoring values
 - For example, we cannot find the mean life expectancy for an income level of 1.5

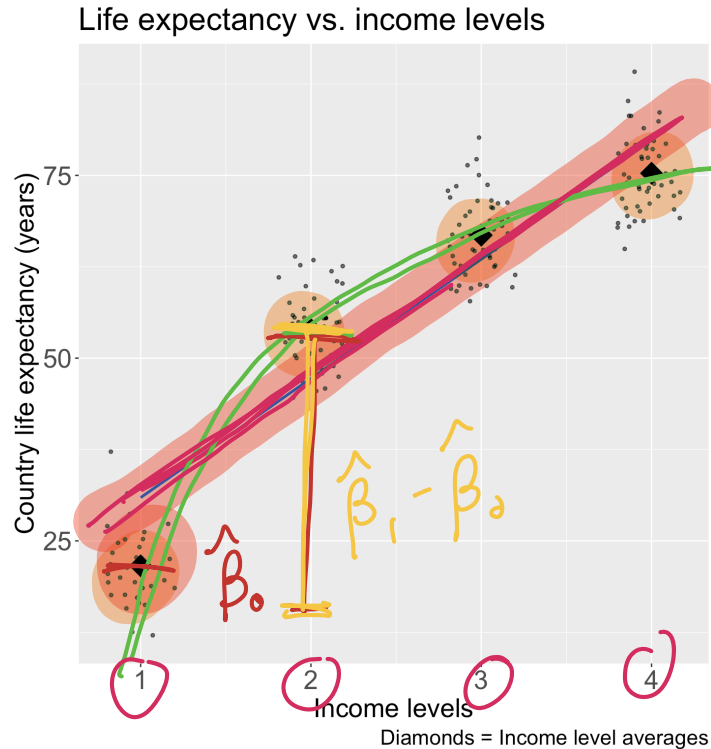
Interpreting the model

term	estimate	std.error	statistic	p.value	conf.low	conf.high
(Intercept)	54.01	1.06	51.03	0.00	51.92	56.10
income_num	6.25	0.37	16.91	0.00	5.52	6.98

$$\widehat{LE} = 54.01 + 6.25 \cdot \text{Income level}$$

- **Interpreting the intercept:** At an income level of 0, mean life expectancy is 54.01 (95% CI: 51.92, 56.10).
 - Note: this does not make sense because there is no income level of 0!
- **Interpreting the coefficient for income:** For every 1-level increase in income level, mean life expectancy increases 6.25 years (95% CI: 5.52, 6.98).

What if life expectancy vs. income level looked like this?



- No longer maintaining the linearity assumption
- Need to use reference cell coding

- We would fit the following model:

$$\text{LE} = \beta_0 + \beta_1 \cdot I(\text{Lower middle income}) + \beta_2 \cdot I(\text{Upper middle income}) + \beta_3 \cdot I(\text{High income}) + \epsilon$$

If time...

Let's walk through categorical variables that have multiple selections

- So each group is not mutually exclusive
- We could make an indicator for each category, but individuals could be a part of multiple categories
- Also, thinking about income levels - can we combine two groups to make one??

