Lesson 12: Interactions, Part 2

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2025-02-19

Learning Objectives

Last time:

1. Define confounders and effect modifiers, and how they interact with the main relationship we model.
2. Interpret the interaction component of a model with **a binary categorical covariate and continuous covariate**, and how the main variable's effect changes.
3. Interpret the interaction component of a model with **a multi-level categorical covariate and continuous covariate**, and how the main variable's effect changes.

This time:

4. Interpret the interaction component of a model with **two categorical covariates**, and how the main variable's effect changes.
5. Interpret the interaction component of a model with **two continuous covariates**, and how the main variable's effect changes.
6. Report results for a best-fit line (with confidence intervals) at different levels of an effect measure modifier

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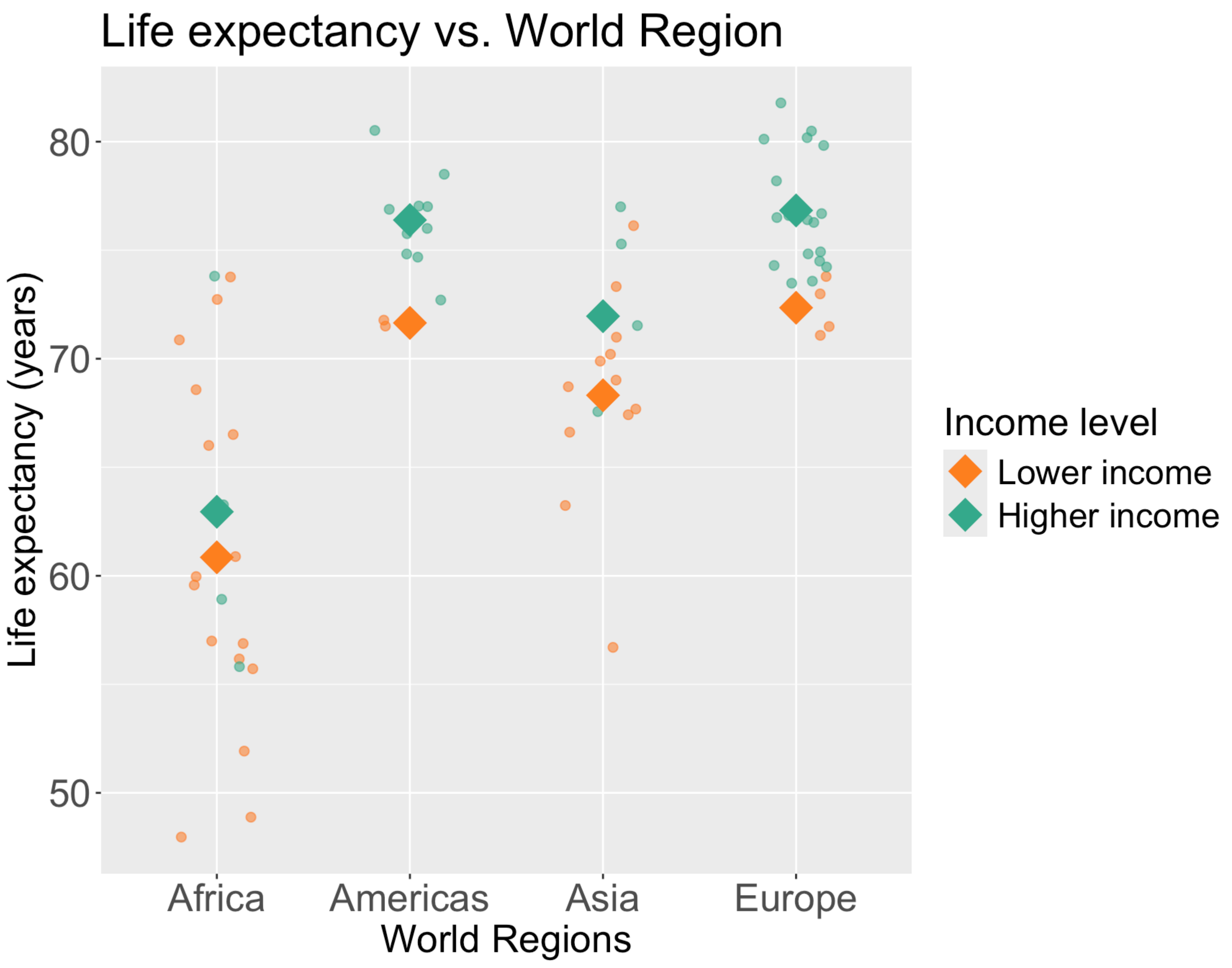
This time:

4. Interpret the interaction component of a model with **two categorical covariates**, and how the main variable's effect changes.
5. Interpret the interaction component of a model with **two continuous covariates**, and how the main variable's effect changes.

Do we think income level can be an effect modifier for world region?

- Taking a break from female literacy rate to demonstrate interactions for two categorical variables
- We can start by visualizing the relationship between life expectancy and world region *by income level*
- Questions of interest: Does the effect of world region on life expectancy differ depending on income level?
  - This is the same as: Is income level an effect modifier for world region?
- Let's run an interaction model to see!



Model with interaction between a *multi-level categorical and binary variables*

Model we are fitting:

$$
\begin{aligned}
LE = & \beta_0 + \beta_1 I(\text{high income}) + \beta_2 I(\text{Americas}) + \beta_3 I(\text{Asia}) + \beta_4 I(\text{Europe}) + \\
& \beta_5 \cdot I(\text{high income}) \cdot I(\text{Americas}) + \beta_6 \cdot I(\text{high income}) \cdot I(\text{Asia}) + \\
& \beta_7 \cdot I(\text{high income}) \cdot I(\text{Europe}) + \epsilon
\end{aligned}
$$

- LE as life expectancy
- $I(\text{high income})$ as indicator of high income
- $I(\text{Americas}), I(\text{Asia}), I(\text{Europe})$ as the indicator for each world region

In R:

```
# gapm_sub = gapm_sub %>% mutate(income_levels2 = relevel(income_levels2, ref = "Hi

m_int_wr_inc = lm(LifeExpectancyYrs ~ income_levels2 + four_regions +
                    income_levels2*four_regions, data = gapm_sub)
m_int_wr_inc = lm(LifeExpectancyYrs ~ income_levels2*four_regions,
                  data = gapm_sub)
```

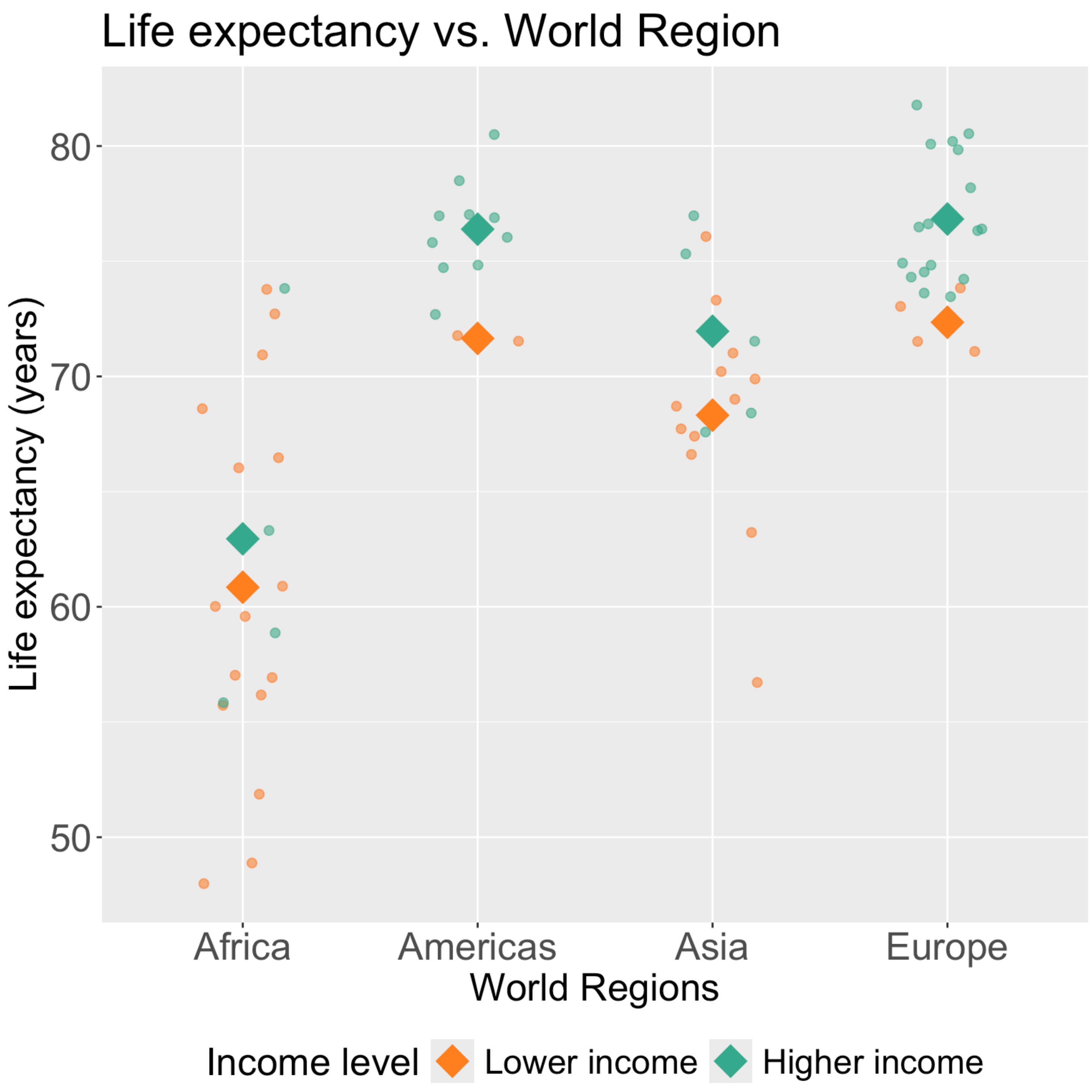
Displaying the regression table and writing fitted regression equation

```
1  tidy(m_int_wr_inc, conf.int=T) %>% gt() %>% tab_options(table.font.size = 25) %>% f
```

term	estimate	std.error	statistic	p.value	conf.low	conf.high
(Intercept)	60.850	1.281	47.488	0.000	58.290	63.410
income_levels2Higher income	2.100	2.865	0.733	0.466	−3.624	7.824
four_regionsAmericas	10.800	3.844	2.810	0.007	3.121	18.479
four_regionsAsia	7.467	1.957	3.815	0.000	3.556	11.377
four_regionsEurope	11.500	2.865	4.014	0.000	5.776	17.224
income_levels2Higher income:four_regionsAmericas	2.640	4.896	0.539	0.592	−7.141	12.421
income_levels2Higher income:four_regionsAsia	1.543	3.956	0.390	0.698	−6.360	9.447
income_levels2Higher income:four_regionsEurope	2.382	4.020	0.592	0.556	−5.649	10.412

$$
\begin{aligned}
\widehat{LE} = & \widehat{\beta}_0 + \widehat{\beta}_1 I(\text{high income}) + \widehat{\beta}_2 I(\text{Americas}) + \widehat{\beta}_3 I(\text{Asia}) + \widehat{\beta}_4 I(\text{Europe}) + \\
& \widehat{\beta}_5 \cdot I(\text{high income}) \cdot I(\text{Americas}) + \widehat{\beta}_6 \cdot I(\text{high income}) \cdot I(\text{Asia}) + \\
& \widehat{\beta}_7 \cdot I(\text{high income}) \cdot I(\text{Europe}) \\
\widehat{LE} = & 60.85 + 2.10 \cdot I(\text{high income}) + 10.8 \cdot I(\text{Americas}) + 7.47 \cdot I(\text{Asia}) + 11.50 \cdot I(\text{Europe}) + \\
& 2.64 \cdot I(\text{high income}) \cdot I(\text{Americas}) + 1.54 \cdot I(\text{high income}) \cdot I(\text{Asia}) + \\
& 2.38 \cdot I(\text{high income}) \cdot I(\text{Europe})
\end{aligned}
$$

Poll Everywhere Question 4



Comparing fitted regression *means* for each world region

$$\begin{aligned}\widehat{LE} =& \widehat{\beta}_0 + \widehat{\beta}_1 I(\text{high income}) + \widehat{\beta}_2 I(\text{Americas}) + \widehat{\beta}_3 I(\text{Asia}) + \widehat{\beta}_4 I(\text{Europe}) + \\ & \widehat{\beta}_5 \cdot I(\text{high income}) \cdot I(\text{Americas}) + \widehat{\beta}_6 \cdot I(\text{high income}) \cdot I(\text{Asia}) + \\ & \widehat{\beta}_7 \cdot I(\text{high income}) \cdot I(\text{Europe}) \\ \widehat{LE} =& 60.85 + 2.10 \cdot I(\text{high income}) + 10.8 \cdot I(\text{Americas}) + 7.47 \cdot I(\text{Asia}) + 11.50 \cdot I(\text{Europe}) + \\ & 2.64 \cdot I(\text{high income}) \cdot I(\text{Americas}) + 1.54 \cdot I(\text{high income}) \cdot I(\text{Asia}) + \\ & 2.38 \cdot I(\text{high income}) \cdot I(\text{Europe})\end{aligned}$$

Africa

$$\begin{aligned}\widehat{LE} =& \widehat{\beta}_0 + \widehat{\beta}_1 I(\text{high income}) + \\ & \widehat{\beta}_2 \cdot 0 + \widehat{\beta}_3 \cdot 0 + \widehat{\beta}_4 \cdot 0 + \\ & \widehat{\beta}_5 I(\text{high income}) \cdot 0 + \\ & \widehat{\beta}_6 I(\text{high income}) \cdot 0 + \\ & \widehat{\beta}_7 I(\text{high income}) \cdot 0 \\ \widehat{LE} =& \widehat{\beta}_0 + \widehat{\beta}_1 I(\text{high income})\end{aligned}$$

The Americas

$$\begin{aligned}\widehat{LE} =& \widehat{\beta}_0 + \widehat{\beta}_1 I(\text{high income}) + \\ & \widehat{\beta}_2 \cdot 1 + \widehat{\beta}_3 \cdot 0 + \widehat{\beta}_4 \cdot 0 + \\ & \widehat{\beta}_5 I(\text{high income}) \cdot 1 + \\ & \widehat{\beta}_6 I(\text{high income}) \cdot 0 + \\ & \widehat{\beta}_7 I(\text{high income}) \cdot 0 \\ \widehat{LE} =& (\widehat{\beta}_0 + \widehat{\beta}_2) + \\ & (\widehat{\beta}_1 + \widehat{\beta}_5) I(\text{high income})\end{aligned}$$

Asia

$$\begin{aligned}\widehat{LE} =& \widehat{\beta}_0 + \widehat{\beta}_1 I(\text{high income}) + \\ & \widehat{\beta}_2 \cdot 0 + \widehat{\beta}_3 \cdot 1 + \widehat{\beta}_4 \cdot 0 + \\ & \widehat{\beta}_5 I(\text{high income}) \cdot 0 + \\ & \widehat{\beta}_6 I(\text{high income}) \cdot 1 + \\ & \widehat{\beta}_7 I(\text{high income}) \cdot 0 \\ \widehat{LE} =& (\widehat{\beta}_0 + \widehat{\beta}_3) + \\ & (\widehat{\beta}_1 + \widehat{\beta}_6) I(\text{high income})\end{aligned}$$

Europe

$$\begin{aligned}\widehat{LE} =& \widehat{\beta}_0 + \widehat{\beta}_1 I(\text{high income}) + \\ & \widehat{\beta}_2 \cdot 0 + \widehat{\beta}_3 \cdot 0 + \widehat{\beta}_4 \cdot 1 + \\ & \widehat{\beta}_5 I(\text{high income}) \cdot 0 + \\ & \widehat{\beta}_6 I(\text{high income}) \cdot 0 + \\ & \widehat{\beta}_7 I(\text{high income}) \cdot 1 \\ \widehat{LE} =& (\widehat{\beta}_0 + \widehat{\beta}_4) + \\ & (\widehat{\beta}_1 + \widehat{\beta}_7) I(\text{high income})\end{aligned}$$

Comparing fitted regression *means* for each income level

$$\begin{aligned}\widehat{LE} =& \widehat{\beta}_0 + \widehat{\beta}_1 I(\text{high income}) + \widehat{\beta}_2 I(\text{Americas}) + \widehat{\beta}_3 I(\text{Asia}) + \widehat{\beta}_4 I(\text{Europe}) + \\ & \widehat{\beta}_5 \cdot I(\text{high income}) \cdot I(\text{Americas}) + \widehat{\beta}_6 \cdot I(\text{high income}) \cdot I(\text{Asia}) + \\ & \widehat{\beta}_7 \cdot I(\text{high income}) \cdot I(\text{Europe}) \\ \widehat{LE} =& 60.85 + 2.10 \cdot I(\text{high income}) + 10.8 \cdot I(\text{Americas}) + 7.47 \cdot I(\text{Asia}) + 11.50 \cdot I(\text{Europe}) + \\ & 2.64 \cdot I(\text{high income}) \cdot I(\text{Americas}) + 1.54 \cdot I(\text{high income}) \cdot I(\text{Asia}) + \\ & 2.38 \cdot I(\text{high income}) \cdot I(\text{Europe})\end{aligned}$$

For lower income countries: $I(\text{high income}) = 0$

$$\begin{aligned}\widehat{LE} =& \widehat{\beta}_0 + \widehat{\beta}_1 \cdot 0 + \widehat{\beta}_2 I(\text{Americas}) + \widehat{\beta}_3 I(\text{Asia}) + \widehat{\beta}_4 I(\text{Europe}) + \\ & \widehat{\beta}_5 \cdot 0 \cdot I(\text{Americas}) + \widehat{\beta}_6 \cdot 0 \cdot I(\text{Asia}) + \widehat{\beta}_7 \cdot 0 \cdot I(\text{Europe}) \\ \widehat{LE} =& \widehat{\beta}_0 + \widehat{\beta}_2 I(\text{Americas}) + \widehat{\beta}_3 I(\text{Asia}) + \widehat{\beta}_4 I(\text{Europe})\end{aligned}$$

For higher income countries: $I(\text{high income}) = 1$

$$\begin{aligned}\widehat{LE} =& \widehat{\beta}_0 + \widehat{\beta}_1 \cdot 1 + \widehat{\beta}_2 I(\text{Americas}) + \widehat{\beta}_3 I(\text{Asia}) + \widehat{\beta}_4 I(\text{Europe}) + \\ & \widehat{\beta}_5 \cdot 1 \cdot I(\text{Americas}) + \widehat{\beta}_6 \cdot 1 \cdot I(\text{Asia}) + \widehat{\beta}_7 \cdot 1 \cdot I(\text{Europe}) \\ \widehat{LE} =& (\widehat{\beta}_0 + \widehat{\beta}_1) + (\widehat{\beta}_2 + \widehat{\beta}_5) I(\text{Americas}) + (\widehat{\beta}_3 + \widehat{\beta}_6) I(\text{Asia}) + \\ & (\widehat{\beta}_4 + \widehat{\beta}_7) I(\text{Europe})\end{aligned}$$

- Example interpretation: The America's effect on mean life expectancy increases $\widehat{\beta}_5$ comparing high income to low income countries.

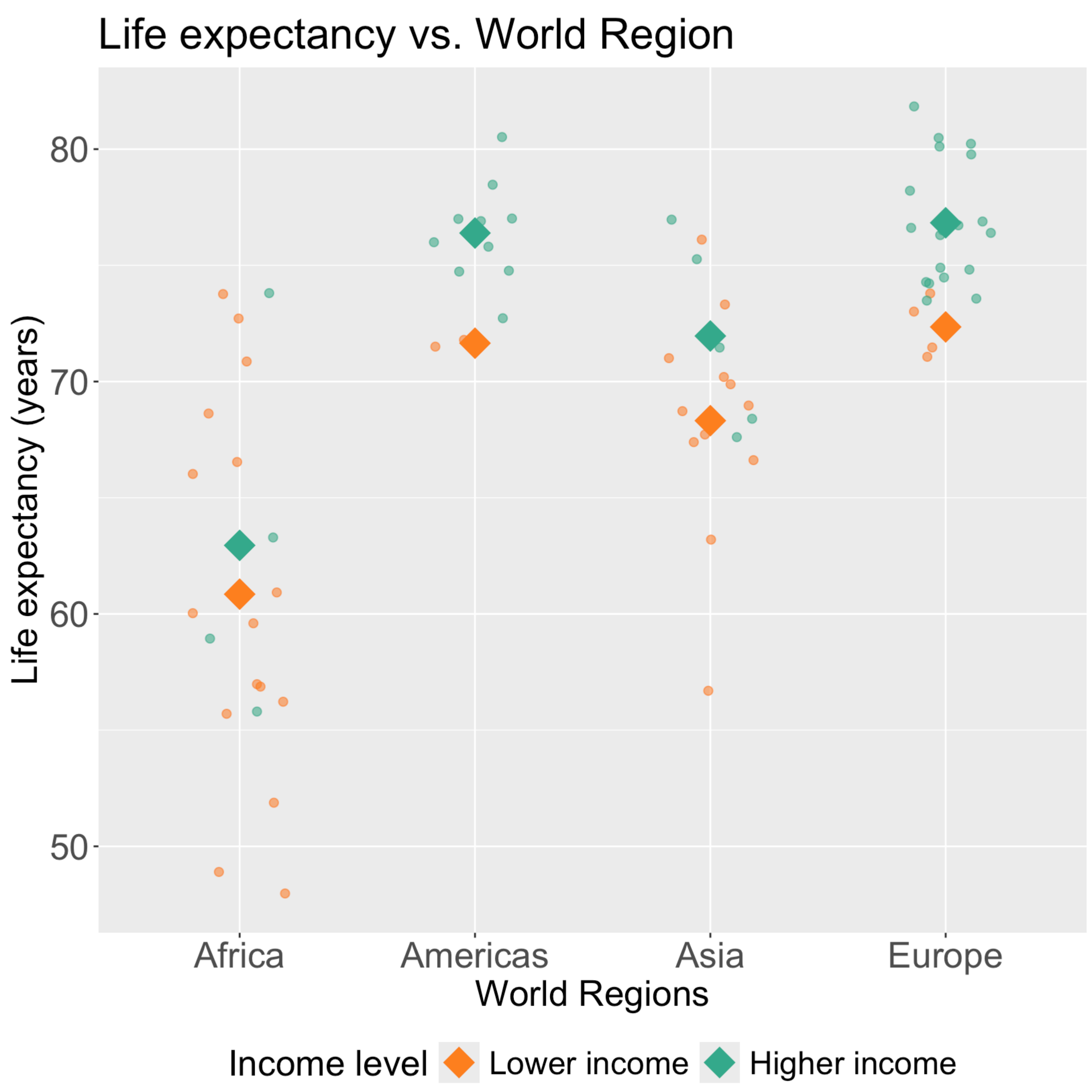
Let's take a look back at the plot

For lower income countries: $I(\text{high income}) = 0$

$$\widehat{LE} = \widehat{\beta}_0 + \widehat{\beta}_2 I(\text{Americas}) + \widehat{\beta}_3 I(\text{Asia}) + \widehat{\beta}_4 I(\text{Europe})$$

For higher income countries: $I(\text{high income}) = 1$

$$\widehat{LE} = (\widehat{\beta}_0 + \widehat{\beta}_1) + (\widehat{\beta}_2 + \widehat{\beta}_5) I(\text{Americas}) + (\widehat{\beta}_3 + \widehat{\beta}_6) I(\text{Asia}) + (\widehat{\beta}_4 + \widehat{\beta}_7) I(\text{Europe})$$



Interpretation for interaction between two categorical variables

$$\begin{aligned}\widehat{LE} =& \widehat{\beta}_0 + \widehat{\beta}_1 \cdot I(\text{high income}) + \widehat{\beta}_2 I(\text{Americas}) + \widehat{\beta}_3 I(\text{Asia}) + \widehat{\beta}_4 I(\text{Europe}) + \\ & \widehat{\beta}_5 \cdot I(\text{high income}) \cdot I(\text{Americas}) + \widehat{\beta}_6 \cdot I(\text{high income}) \cdot I(\text{Asia}) + \\ & \widehat{\beta}_7 \cdot I(\text{high income}) \cdot I(\text{Europe})\end{aligned}$$

$$\begin{aligned}\widehat{LE} =& \left[\widehat{\beta}_0 + \widehat{\beta}_1 \cdot I(\text{high income})\right] + \left[\widehat{\beta}_2 + \widehat{\beta}_5 \cdot I(\text{high income})\right] I(\text{Americas}) + \\ & \left[\widehat{\beta}_3 + \widehat{\beta}_6 \cdot I(\text{high income})\right] I(\text{Asia}) + \left[\widehat{\beta}_4 + \widehat{\beta}_7 \cdot I(\text{high income})\right] I(\text{Europe})\end{aligned}$$

- Interpretation:
  - β_1 = mean change in the Africa's life expectancy, comparing high income to low income countries
  - β_5 = mean change in the Americas' effect, comparing high income to low income countries
  - β_6 = mean change in Asia's effect, comparing high income to low income countries
  - β_7 = mean change in Europe's effect, comparing high income to low income countries

Test interaction between two categorical variables (1/2)

- We run an F-test for a group of coefficients (β_5, β_6, β_7) in the below model (see lesson 9)

$$\begin{aligned} LE = & \beta_0 + \beta_1 I(\text{high income}) + \beta_2 I(\text{Americas}) + \beta_3 I(\text{Asia}) + \beta_4 I(\text{Europe}) + \\ & \beta_5 \cdot I(\text{high income}) \cdot I(\text{Americas}) + \beta_6 \cdot I(\text{high income}) \cdot I(\text{Asia}) + \\ & \beta_7 \cdot I(\text{high income}) \cdot I(\text{Europe}) + \epsilon \end{aligned}$$

Null H_0

$\beta_5 = \beta_6 = \beta_7 = 0$

Alternative H_1

$\beta_5 \neq 0$ and/or $\beta_6 \neq 0$ and/or $\beta_7 \neq 0$

Null / Smaller / Reduced model

$$\begin{aligned} LE = & \beta_0 + \beta_1 I(\text{high income}) + \beta_2 I(\text{Americas}) + \\ & \beta_3 I(\text{Asia}) + \beta_4 I(\text{Europe}) + \epsilon \end{aligned}$$

Alternative / Larger / Full model

$$\begin{aligned} LE = & \beta_0 + \beta_1 I(\text{high income}) + \beta_2 I(\text{Americas}) + \beta_3 I(\text{Asia}) + \\ & \beta_4 I(\text{Europe}) + \beta_5 \cdot I(\text{high income}) \cdot I(\text{Americas}) + \\ & \beta_6 \cdot I(\text{high income}) \cdot I(\text{Asia}) + \beta_7 \cdot I(\text{high income}) \cdot I(\text{Europe}) + \epsilon \end{aligned}$$

Test interaction between two categorical variables (2/2)

- Fit the reduced and full model

```
m_int_wr_inc_red = lm(LifeExpectancyYrs ~ income_levels2 + four_regions,
                       data = gapm_sub)
m_int_wr_inc_full = lm(LifeExpectancyYrs ~ income_levels2 + four_regions +
                              income_levels2*four_regions, data = gapm_sub)
```

- Display the ANOVA table with F-statistic and p-value

term	df.residual	rss	df	sumsq	statistic	p.value
LifeExpectancyYrs ~ income_levels2 + four_regions	67.000	1,693.242	NA	NA	NA	NA
LifeExpectancyYrs ~ income_levels2 + four_regions + income_levels2 * four_regions	64.000	1,681.304	3.000	11.938	0.151	0.928

- Conclusion: There is not a significant interaction between world region and income level (p = 0.928).

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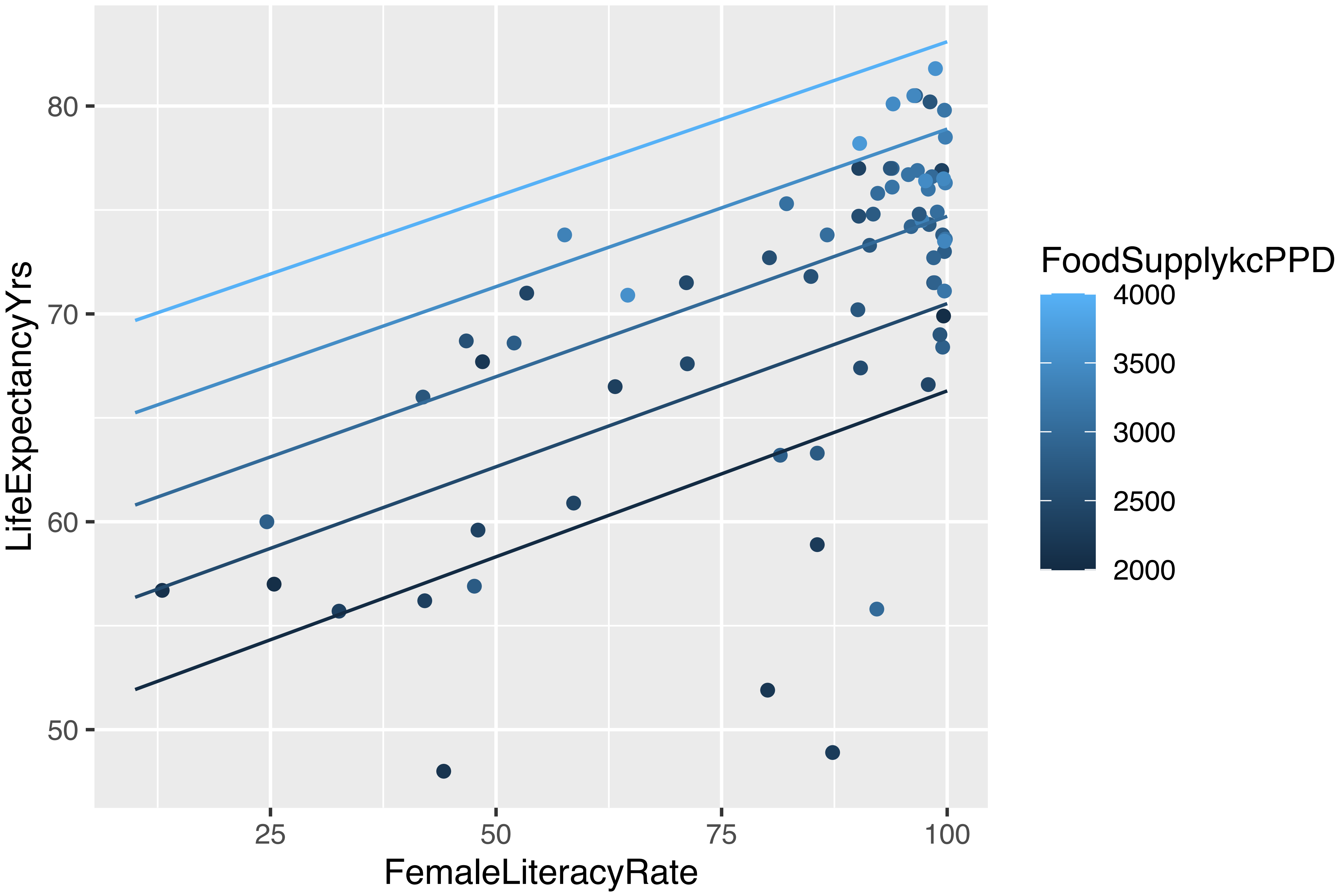
3. Interpret the interaction component of a model with **a multi-level categorical covariate and continuous covariate**, and how the main variable's effect changes.

This time:

4. Interpret the interaction component of a model with **two categorical covariates**, and how the main variable's effect changes.
5. Interpret the interaction component of a model with **two continuous covariates**, and how the main variable's effect changes.

Do we think food supply is an effect modifier for female literacy rate?

- We can start by visualizing the relationship between life expectancy and female literacy rate *by food supply*
- Questions of interest: Does the effect of female literacy rate on life expectancy differ depending on food supply?
  - This is the same as: Is food supply is an effect modifier for female literacy rate? Is food supply an effect modifier of the association between life expectancy and female literacy rate?
- Let's run an interaction model to see!



Model with interaction between *two continuous variables*

Model we are fitting:

$$LE = \beta_0 + \beta_1 FLR^c + \beta_2 FS^c + \beta_3 FLR^c \cdot FS^c + \epsilon$$

- LE as life expectancy
- FLR^c as the **centered** around the mean female literacy rate (continuous variable)
- FS^c as the **centered** around the mean food supply (continuous variable)

▶ Code to center FLR and FS

In R:

```
m_int_fs = lm(LifeExpectancyYrs ~ FLR_c + FS_c + FLR_c*FS_c, data = gapm_sub)
```

OR

```
m_int_fs = lm(LifeExpectancyYrs ~ FLR_c*FS_c, data = gapm_sub)
```

Displaying the regression table and writing fitted regression equation

```r
tidy_m_fs = tidy(m_int_fs, conf.int=T)
tidy_m_fs %>% gt() %>% tab_options(table.font.size = 35) %>% fmt_number(decimals =
```

term	estimate	std.error	statistic	p.value	conf.low	conf.high
(Intercept)	70.32060	0.72393	97.13721	0.00000	68.87601	71.76518
FLR_c	0.15532	0.03808	4.07905	0.00012	0.07934	0.23130
FS_c	0.00849	0.00182	4.67908	0.00001	0.00487	0.01212
FLR_c:FS_c	−0.00001	0.00008	−0.06908	0.94513	−0.00016	0.00015

$$\widehat{LE} = \widehat{\beta}_0 + \widehat{\beta}_1 FLR^c + \widehat{\beta}_2 FS^c + \widehat{\beta}_3 FLR^c \cdot FS^c$$

$$\widehat{LE} = 70.32 + 0.16 \cdot FLR^c + 0.01 \cdot FS^c - 0.00001 \cdot FLR^c \cdot FS^c$$

Comparing fitted regression lines for various food supply values

$$\widehat{LE} = \widehat{\beta}_0 + \widehat{\beta}_1 FLR^c + \widehat{\beta}_2 FS^c + \widehat{\beta}_3 FLR^c \cdot FS^c$$
$$\widehat{LE} = 70.32 + 0.16 \cdot FLR^c + 0.01 \cdot FS^c - 0.00001 \cdot FLR^c \cdot FS^c$$

To identify different lines, we need to pick example values of Food Supply:

Food Supply of 1812 kcal PPD

$$\begin{aligned}\widehat{LE} =& \widehat{\beta}_0 + \widehat{\beta}_1 FLR^c + \\ & \widehat{\beta}_2 \cdot (-1000) + \\ & \widehat{\beta}_3 FLR^c \cdot (-1000) \\ \widehat{LE} =& (\widehat{\beta}_0 - 1000\widehat{\beta}_2) + \\ & (\widehat{\beta}_1 - 1000\widehat{\beta}_3) FLR^c\end{aligned}$$

Food Supply of 2812 kcal PPD

$$\begin{aligned}\widehat{LE} =& \widehat{\beta}_0 + \widehat{\beta}_1 FLR^c + \\ & \widehat{\beta}_2 \cdot 0 + \\ & \widehat{\beta}_3 FLR^c \cdot 0 \\ \widehat{LE} =& (\widehat{\beta}_0) + \\ & (\widehat{\beta}_1) FLR^c\end{aligned}$$

Food Supply of 3812 kcal PPD

$$\begin{aligned}\widehat{LE} =& \widehat{\beta}_0 + \widehat{\beta}_1 FLR^c + \\ & \widehat{\beta}_2 \cdot 1000 + \\ & \widehat{\beta}_3 FLR^c \cdot 1000 \\ \widehat{LE} =& (\widehat{\beta}_0 + 1000\widehat{\beta}_2) + \\ & (\widehat{\beta}_1 + 1000\widehat{\beta}_3) FLR^c\end{aligned}$$

Poll Everywhere Question??

Interpretation for interaction between two continuous variables

$$\widehat{LE} = \widehat{\beta}_0 + \widehat{\beta}_1 FLR^c + \widehat{\beta}_2 FS^c + \widehat{\beta}_3 FLR^c \cdot FS^c$$
$$\widehat{LE} = \left[\widehat{\beta}_0 + \widehat{\beta}_2 \cdot FS^c\right] + \underbrace{\left[\widehat{\beta}_1 + \widehat{\beta}_3 \cdot FS^c\right]}_{\text{FLR's effect}} FLR$$

- Interpretation:
  - β_3 = mean change in female literacy rate's effect, for every one kcal PPD increase in food supply
- In summary, the interaction term can be interpreted as "difference in adjusted female literacy rate effect for every 1 kcal PPD increase in food supply"
- It will be helpful to test the interaction to round out this interpretation!!

Test interaction between two continuous variables

- We run an F-test for a single coefficients (β_3) in the below model (see lesson 9)

$$LE = \beta_0 + \beta_1 FLR^c + \beta_2 FS^c + \beta_3 FLR^c \cdot FS^c + \epsilon$$

Null H_0

$$\beta_3 = 0$$

Alternative H_1

$$\beta_3 \neq 0$$

Null / Smaller / Reduced model

$$LE = \beta_0 + \beta_1 FLR^c + \beta_2 FS^c + \epsilon$$

Alternative / Larger / Full model

$$\begin{aligned} LE = &\beta_0 + \beta_1 FLR^c + \beta_2 FS^c + \\ &\beta_3 FLR^c \cdot FS^c + \epsilon \end{aligned}$$

Test interaction between two continuous variables

- Fit the reduced and full model

```
m_int_fs_red = lm(LifeExpectancyYrs ~ FLR_c + FS_c,
                   data = gapm_sub)
m_int_fs_full = lm(LifeExpectancyYrs ~ FLR_c + FS_c +
                  FLR_c*FS_c, data = gapm_sub)
```

▶ Display the ANOVA table with F-statistic and p-value

term	df.residual	rss	df	sumsq	statistic	p.value
LifeExpectancyYrs ~ FLR_c + FS_c	69.000	2,005.556	NA	NA	NA	NA
LifeExpectancyYrs ~ FLR_c + FS_c + FLR_c * FS_c	68.000	2,005.415	1.000	0.141	0.005	0.945

- Conclusion: There is not a significant interaction between female literacy rate and food supply (p = 0.945). Food supply is not an effect modifier of the association between female literacy rate and life expectancy.

Learning Objective

Bonus learning objective that's not really bonus but just a last minute addition

6. Report results for a best-fit line (with confidence intervals) at different levels of an effect measure modifier

How to find the confidence interval for each slope?

- In the example with FS and FLR, we showed:

Best-fit line for Food Supply of 3812 kcal PPD

$$\widehat{LE} = \left(\widehat{\beta}_0 + 1000\widehat{\beta}_2\right) + \left(\widehat{\beta}_1 + 1000\widehat{\beta}_3\right)FLR^c$$

- Often, we want to report the estimate of the combined coefficients: $\widehat{\beta}_1 + 1000\widehat{\beta}_3$
  - This allows us to make a statement like: "At a food supply of 3812 kcal PPD, mean life expectancy increases ($\widehat{\beta}_1 + 1000\widehat{\beta}_3$) years for every one percent increase in female literacy rate (95% CI: __, __)."

- We can calculate $\widehat{\beta}_1 + 1000\widehat{\beta}_3$ by using the values of the estimated coefficients
- BUT we always want to have a **95% confidence interval** when we report this combined estimate!!

Getting a 95% confidence interval requires linear combinations!

- If we want a confidence interval for $\widehat{\beta}_1 + 1000\widehat{\beta}_3$, then we would use the formula:

$$\left(\widehat{\beta}_1 + 1000\widehat{\beta}_3\right) \pm t^* \times SE_{(\beta_1+1000\beta_3)}$$

- The hard part is figuring out what $SE_{(\beta_1+1000\beta_3)}$ (or $\mathrm{Var}(\beta_1 + 1000\beta_3)$) equals
- We need to go back to variance of linear combinations (BSTA 512/612, EPI 525):

$$\mathrm{Var}(aX + bY) = a^2\mathrm{Var}(X) + b^2\mathrm{Var}(Y) + 2ab\mathrm{Cov}(X, Y)$$

or

$$\mathrm{Var}(aX - bY) = a^2\mathrm{Var}(X) + b^2\mathrm{Var}(Y) - 2ab\mathrm{Cov}(X, Y)$$

Reference: calculating $SE_{(\beta_1+1000\beta_3)}$ by hand

- A helpful function that returns the variance-covariance matric of all the coefficients in model `m_int_fs`:

```r
vcov(m_int_fs)
```

```
              (Intercept)         FLR_c         FS_c    FLR_c:FS_c
(Intercept)  5.240754e-01 -6.771205e-03  5.586960e-05 -2.609611e-05
FLR_c       -6.771205e-03  1.449828e-03 -3.150719e-05  1.543619e-06
FS_c         5.586960e-05 -3.150719e-05  3.294981e-06 -1.273649e-08
FLR_c:FS_c  -2.609611e-05  1.543619e-06 -1.273649e-08  5.949082e-09
```

$$\mathrm{Var}(\beta_1) = 0.0014498$$
$$\mathrm{Var}(\beta_3) = 6 \times 10^{-9}$$
$$\mathrm{Cov}(\beta_1, \beta_3) = 1.544 \times 10^{-6}$$

$$\mathrm{Var}(\beta_1 + 1000\beta_3) = \mathrm{Var}(\beta_1) + 1000^2\mathrm{Var}(\beta_3) + 2000\mathrm{Cov}(\beta_1, \beta_3)$$
$$\mathrm{Var}(\beta_1 + 1000\beta_3) = 0.0014498 + 1000^2 \times 6 \times 10^{-9} + 2000 \times 1.544 \times 10^{-6}$$
$$\mathrm{Var}(\beta_1 + 1000\beta_3) = 0.0104861$$
$$SE_{(\beta_1+1000\beta_3)} = \sqrt{0.0104861}$$
$$SE_{(\beta_1+1000\beta_3)} = 0.1024019$$

We can use R and `estimable()` to find the estimate and CI

For $\widehat{\beta}_1 + 1000\widehat{\beta}_3$:

```r
library(gmodels)
m_int_fs %>% estimable(
                     c("(Intercept)" = 0,      # beta0
                       "FLR_c"       = 1,      # beta1
                       "FS_c"        = 0,      # beta2
                       "FLR_c:FS_c"  = 1000),  # beta3
                     conf.int = 0.95)
```

```
              Estimate Std. Error  t value DF  Pr(>|t|)    Lower.CI  Upper.CI
(0 1 0 1000) 0.1499879  0.1024019 1.464698 68 0.1476115 -0.05435192 0.3543277
```

Our conclusion: At a food supply of 3812 kcal PPD, mean life expectancy increases 0.14999 years for every one percent increase in female literacy rate (95% CI: -0.05435, 0.35433).

Another example: income (binary) and FLR (1/2)

```
m_int_inc2 = gapm_sub %>%
  lm(formula = LifeExpectancyYrs ~ FLR_c*income_levels2)
```

$$\widehat{LE} = \widehat{\beta}_0 + \widehat{\beta}_1 FLR + \widehat{\beta}_2 I(\text{high income}) + \widehat{\beta}_3 FLR \cdot I(\text{high income})$$
$$\widehat{LE} = 54.85 + 0.156 \cdot FLR - 16.65 \cdot I(\text{high income}) + 0.228 \cdot FLR \cdot I(\text{high income})$$

For lower income countries: $I(\text{high income}) = 0$

$$\widehat{LE} = \widehat{\beta}_0 + \widehat{\beta}_1 FLR + \widehat{\beta}_2 \cdot 0 + \widehat{\beta}_3 FLR \cdot 0$$
$$\widehat{LE} = 54.85 + 0.156 \cdot FLR - 16.65 \cdot 0 + 0.228 \cdot FLR \cdot 0$$
$$\widehat{LE} = 54.85 + 0.156 \cdot FLR$$

For higher income countries: $I(\text{high income}) = 1$

$$\widehat{LE} = \widehat{\beta}_0 + \widehat{\beta}_1 FLR + \widehat{\beta}_2 \cdot 1 + \widehat{\beta}_3 FLR \cdot 1$$
$$\widehat{LE} = 54.85 + 0.156 \cdot FLR - 16.65 \cdot 1 + 0.228 \cdot FLR \cdot 1$$
$$\widehat{LE} = 38.2 + 0.384 \cdot FLR$$

Another example: income (binary) and FLR (2/2)

```
m_int_inc2$coefficients # I just need to see the exact names
```

```
                      (Intercept)                                 FLR_c 
                       67.6818102                             0.1564398 
      income_levels2Higher income FLR_c:income_levels2Higher income 
                        2.0729925                             0.2282290 
```

```
m_int_inc2 %>% estimable(
                    c("(Intercept)" = 0,       # beta0
                      "FLR_c"       = 1,       # beta1
                      "income_levels2Higher income" = 0,       # beta2
                      "FLR_c:income_levels2Higher income"  = 1),   # beta3
                    conf.int = 0.95)
```

```
           Estimate Std. Error  t value DF   Pr(>|t|)   Lower.CI  Upper.CI
(0 1 0 1) 0.3846688  0.1591843 2.416499 68 0.01836001 0.06702138 0.7023161
```

Our conclusion: For countries with high income, mean life expectancy increases 0.385 years for every one percent increase in female literacy rate (95% CI: 0.067, 0.702).

If our example had an effect measure modifier

- None of our examples had a significant interaction, so it's hard to demonstrate exactly how we would report this
- Let's say, **just for example**, that income had a significant interaction with FLR
  - How would we report this to an audience??
- Here's how to report on an interaction/EMM:
  - We found that a country's income status (high or low) is a significant effect measure modifier on female literacy rate (*include p-value for interaction test here*). For countries with high income, mean life expectancy increases 0.385 years for every one percent increase in female literacy rate (95% CI: 0.067, 0.702). For countries with low income, mean life expectancy increases 2.073 years for every one percent increase in female literacy rate (95% CI: -2.922, 7.068)."

Extra Reference Material

General interpretation of the interaction term (reference)

$$E[Y \mid X_1, X_2] = \beta_0 + \underbrace{(\beta_1 + \beta_3 X_2)}_{X_1\text{'s effect}} X_1 + \underbrace{\beta_2 X_2}_{X_2 \text{ held constant}}$$

$$= \beta_0 + \underbrace{(\beta_2 + \beta_3 X_1)}_{X_2\text{'s effect}} X_2 + \underbrace{\beta_1 X_1}_{X_1 \text{ held constant}}$$

- Interpretation:
  - β_3 = mean change in X_1's effect, per unit increase in X_2;
  - = mean change in X_2's effect, per unit increase in X_1;
  - where the "X_1 effect" equals the change in $E[Y]$ per unit increase in X_1 with X_2 held constant, i.e. "adjusted X_1 effect"
- In summary, the interaction term can be interpreted as "difference in adjusted X_1 (or X_2) effect per unit increase in X_2 (or X_1)"

A glimpse at how interactions might be incorporated into model selection

1. Identify outcome (Y) and primary explanatory (X) variables
2. Decide which other variables might be important and could be potential confounders. Add these to the model.
   - This is often done by indentifying variables that previous research deemed important, or researchers believe could be important
   - From a statistical perspective, we often include variables that are significantly associated with the outcome (in their respective SLR)
3. (Optional step) Test 3 way interactions
   - This makes our model incredibly hard to interpret. Our class will not cover this!!
   - We will skip to testing 2 way interactions
4. Test 2 way interactions
   - When testing a 2 way interaction, make sure the full and reduced models contain the main effects
   - First test all the 2 way interactions together using a partial F-test (with $alpha = 0.10$)
     - If this test not significant, do not test 2-way interactions individually
     - If partial F-test is significant, then test each of the 2-way interactions
5. Remaining main effects - to include of not to include?
   - For variables that are included in any interactions, they will be automatically included as main effects and thus not checked for confounding
   - For variables that are not included in any interactions:
     - Check to see if they are confounders by seeing whether exclusion of the variable(s) changes any of the coefficient of the primary explanatory variable (including interactions) X by more than 10%
       - If any of X’s coefficients change when removing the potential confounder, then keep it in the model