Lesson 14: Purposeful model selection

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Learning Objectives

1. Understand the overall steps for purposeful selection as a model building strategy
2. Apply purposeful selection to a dataset using R
3. Use different approaches to assess the linear scale of continuous variables in logistic regression

Regression analysis process

Model Selection

- Building a model
- Selecting variables
- Prediction vs interpretation
- Comparing potential models

Model Fitting

- Find best fit line
- Using OLS in this class
- Parameter estimation
- Categorical covariates
- Interactions

Model Evaluation

- Evaluation of model fit
- Testing model assumptions
- Residuals
- Transformations
- Influential points
- Multicollinearity

Model Use (Inference)

- Inference for coefficients
- Hypothesis testing for coefficients
- Inference for expected Y given X
- Prediction of new Y given X

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"Successful modeling of a complex data set is part science, part statistical methods, and part experience and common sense."

Hosmer, Lemeshow, and Sturdivant Textbook, pg. 101

Overall Process

0. Exploratory data analysis
1. Check unadjusted associations in simple linear regression
2. Enter all covariates in model that meet some threshold
   - One textbook suggest $p < 0.2$ or $p < 0.25$: great for modest sized datasets
   - PLEASE keep in mind sample size in your study
   - Can also use magnitude of association rather than, or along with, p-value
3. Remove those that no longer reach some threshold
   - Compare magnitude of associations to unadjusted version (univariable)
4. Check scaling of continuous and coding of categorical covariates
5. Check for interactions
6. Assess model fit
   - Model assumptions, diagnostics, overall fit

Process with snappier step names

Pre-step: Exploratory data analysis (EDA)

Step 1: Simple linear regressions / analysis

Step 2: Preliminary variable selection

Step 3: Assess change in coefficients

Step 4: Assess scale for continuous variables

Step 5: Check for interactions

Step 6: Assess model fit

Learning Objectives

1. Understand the overall steps for purposeful selection as a model building strategy

2. Apply purposeful selection to a dataset using R

3. Use different approaches to assess the linear scale of continuous variables in logistic regression

Pre-step: Exploratory data analysis

- The following slides are all reference until we get to Step 1
- We have covered exploratory data analysis in other classes and have completed it in our previous labs

Pre-step: Exploratory data analysis

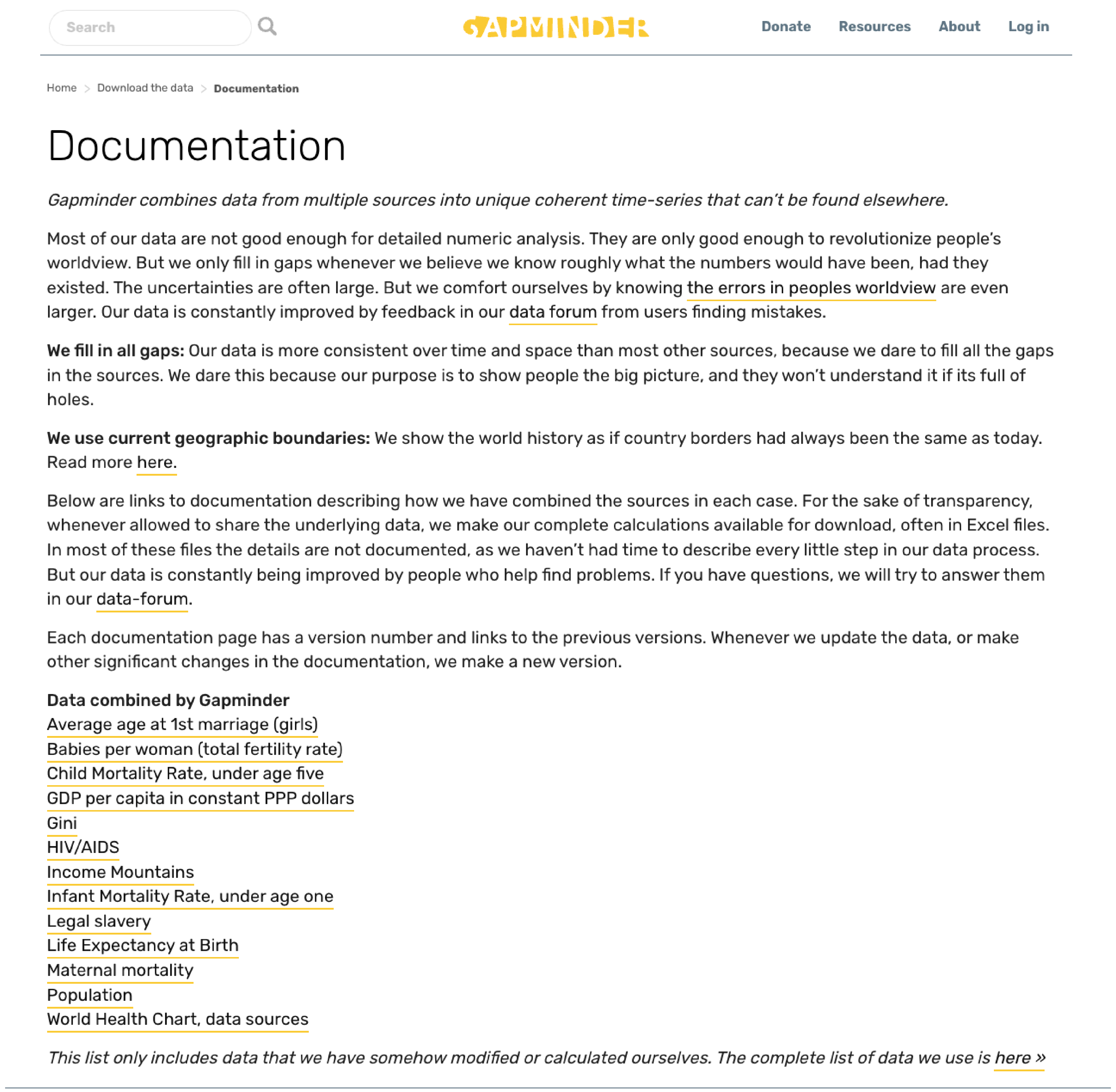
- Things we have been doing over the quarter in class and in our project
- I will not discuss some of the methods mentioned in our lab and data management class
  - I am only going to introduce additional exploratory functions

A few things we can do:

- Check the data
- Study your variables
- Missing data?
- Explore simple relationships and assumptions

Pre-step: Exploratory data analysis: Check the data

- Get to know the potential values for the data
  - Categories
  - Units
- Then make sure the summary of values makes sense
  - If minimum or maximum look outside appropriate range
  - For example: a negative value for a measurement that is inherently positive (like population or income)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Documentation

Gapminder combines data from multiple sources into unique coherent time-series that can't be found elsewhere.

Most of our data are not good enough for detailed numeric analysis. They are only good enough to revolutionize people's worldview. But we only fill in gaps whenever we believe we know roughly what the numbers would have been, had they existed. The uncertainties are often large. But we comfort ourselves by knowing the errors in peoples worldview are even larger. Our data is constantly improved by feedback in our data forum from users finding mistakes.

We fill in all gaps: Our data is more consistent over time and space than most other sources, because we dare to fill all the gaps in the sources. We dare this because our purpose is to show people the big picture, and they won't understand it if its full of holes.

We use current geographic boundaries: We show the world history as if country borders had always been the same as today. Read more here.

Below are links to documentation describing how we have combined the sources in each case. For the sake of transparency, whenever allowed to share the underlying data, we make our complete calculations available for download, often in Excel files. In most of these files the details are not documented, as we haven't had time to describe every little step in our data process. But our data is constantly being improved by people who help find problems. If you have questions, we will try to answer them in our data-forum.

Each documentation page has a version number and links to the previous versions. Whenever we update the data, or make other significant changes in the documentation, we make a new version.

Data combined by Gapminder
Average age at 1st marriage (girls)
Babies per woman (total fertility rate)
Child Mortality Rate, under age five
GDP per capita in constant PPP dollars
Gini
HIV/AIDS
Income Mountains
Infant Mortality Rate, under age one
Legal slavery
Life Expectancy at Birth
Maternal mortality
Population
World Health Chart, data sources

This list only includes data that we have somehow modified or calculated ourselves. The complete list of data we use is here »

https://www.gapminder.org/data/documentation/

Pre-step: Exploratory data analysis: Check the data

- Look at a summary for the raw data
- Typical use:

```
library(skimr)
skim(gapm)
```

- Some `skim()` help

Pre-step: Exploratory data analysis: Check the data

- Look at a summary for the raw data
- Typical use:

```
library(skimr)
skim(gapm)
```

- Some `skim()` help
- Note that `skim(gapm)` looks different because I had to create factors
- I am breaking down the `skim()` function into the categorical and continuous variables only because I want to show them on the slides

```
skim(gapm_sub1) %>% yank("factor")
```

Variable type: factor

skim_variable	n_missing	complete_rate	ordered	n_unique	top_counts
four_regions	0	1.00	FALSE	4	Asi: 57, Afr: 54, Eur: 49, Ame: 35
income_levels1	1	0.99	FALSE	4	Hig: 56, Upp: 55, Low: 52, Low: 31
income_levels2	1	0.99	FALSE	2	Hig: 111, Low: 83

Pre-step: Exploratory data analysis: Check the data

```
skim(gapm_sub1) %>% yank("numeric")
```

Variable type: numeric

skim_variable	n_missing	complete_rate	mean	sd	p0	p25	p50	p75	p100	hist
CO2emissions	4	0.98	4.55	6.10	0.03	0.64	2.41	6.22	41.20	
ElectricityUsePP	58	0.70	4220.92	5964.07	31.10	699.00	2410.00	5600.00	52400.00	
FoodSupplykcPPD	27	0.86	2825.06	443.59	1910.00	2490.00	2775.00	3172.50	3740.00	
IncomePP	2	0.99	16704.45	19098.61	614.00	3370.00	10100.00	22700.00	129000.00	
LifeExpectancyYrs	8	0.96	70.66	8.44	47.50	64.30	72.70	76.90	82.90	
FemaleLiteracyRate	115	0.41	81.65	21.95	13.00	70.97	91.60	98.03	99.80	
WaterSourcePrct	1	0.99	84.84	18.64	18.30	74.90	93.50	99.07	100.00	
Latitude	0	1.00	19.11	23.93	-42.00	4.00	17.33	40.00	65.00	
Longitude	0	1.00	21.98	66.52	-175.00	-5.75	21.00	49.27	179.14	
population_mill	0	1.00	35.95	136.87	0.00	1.73	7.57	24.50	1370.00	

Poll Everywhere Question 1

In the following skim() output, what summaries might you flag?

```
1 skim(gapm_sub1) %>% yank("numeric")
```

Variable type: numeric

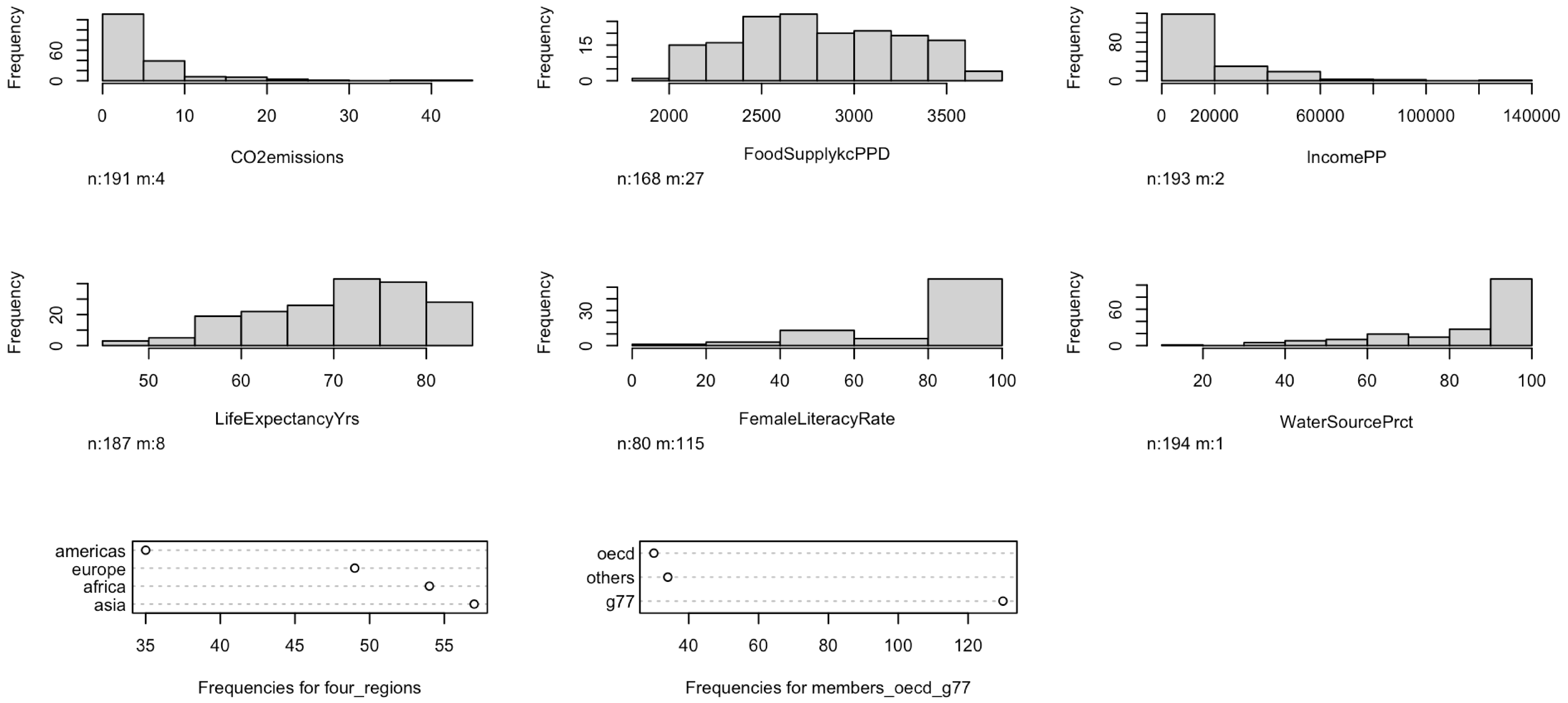
skim_variable	n_missing	complete_rate	mean	sd	p0	p25	p50	p75	p100	hist
CO2emissions	4	0.98	4.55	6.10	0.03	0.64	2.41	6.22	41.20	
ElectricityUsePP	58	0.70	4220.92	5964.07	31.10	699.00	2410.00	5600.00	52400.00	
FoodSupplykcPPD	27	0.86	2825.06	443.59	1910.00	2490.00	2775.00	3172.50	3740.00	
IncomePP	2	0.99	16704.45	19098.61	614.00	3370.00	10100.00	22700.00	129000.00	
LifeExpectancyYrs	8	0.96	70.66	8.44	47.50	64.30	72.70	76.90	82.90	
FemaleLiteracyRate	115	0.41	81.65	21.95	13.00	70.97	91.60	98.03	99.80	
WaterSourcePrct	1	0.99	84.84	18.64	18.30	74.90	93.50	99.07	100.00	
Latitude	0	1.00	19.11	23.93	-42.00	4.00	17.33	40.00	65.00	
Longitude	0	1.00	21.98	66.52	-175.00	-5.75	21.00	49.27	179.14	
population_mill	0	1.00	35.95	136.87	0.00	1.73	7.57	24.50	1370.00	

Pre-step: Exploratory data analysis: Study your variables

- Started this a little bit in previous slide (`skim()`), but you may want to look at things like:
  - Sample size
  - Counts of missing data
  - Means and standard deviations
  - IQRs
  - Medians
  - Minimums and maximums
- Can also look at visuals
  - Continuous variables: histograms (in `skimr() a little)
  - Categorical variables: frequency plots

Pre-step: Exploratory data analysis: Study your variables

```r
library(Hmisc)
hist.data.frame(gapm %>% select(-Longitude, -Latitude, -eight_regions, -six_regions, -geo, -`World bank, 4 income groups 2017`, -country, -population, -`World bank region`, -ElectricityUsePP))
```



Poll Everywhere Question 2

Question: What function might you use to visualize or summarize the frequencies of categorical variables?

Pre-step: Exploratory data analysis: Missing data

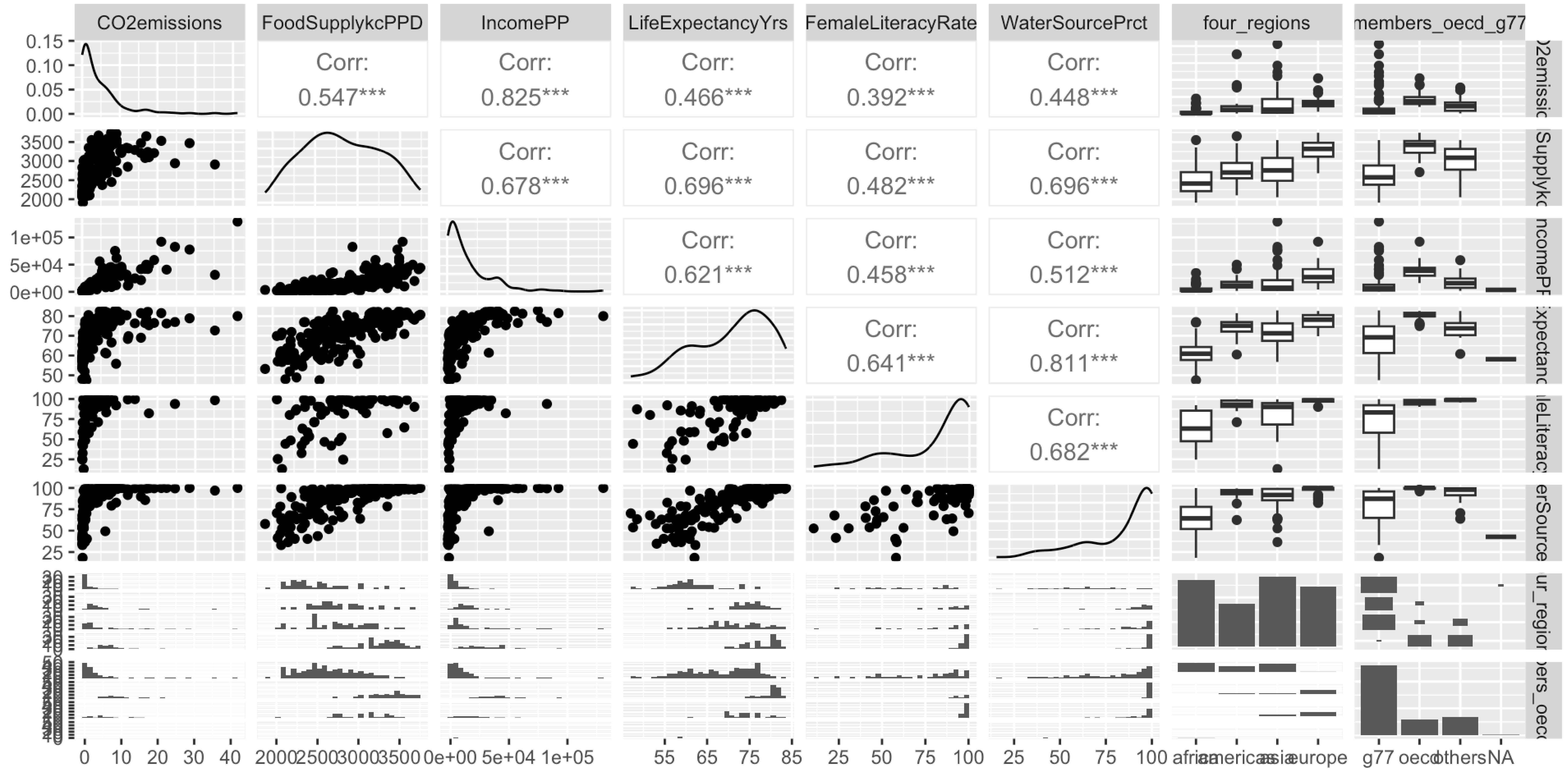
- Why are there missing data?
- Which variables and observations should be excluded because of missing data?
- Will I impute missing data?

- Unfortunately, we don't have time to discuss missing data more thoroughly
- I will try to cover this topic more thoroughly in BSTA 513

- For the Gapminder dataset, we chose to use complete cases

Pre-step / Step 1 : Explore simple relationships and assumptions

```r
gapm2 %>% ggpairs() # gapm2 is a new dataset with some variables selected
```



Poll Everywhere Question 3

Within the ggpairs output, is there a specific row/column that might be helpful when thinking about our outcome?

0

- CO2emissions
- FoodSupplykcPPD
- IncomePP
- LifeExpectancyYrs
- FemaleLiteracyRate
- WaterSourcePrct
- four_regions
- members_oecd_g77

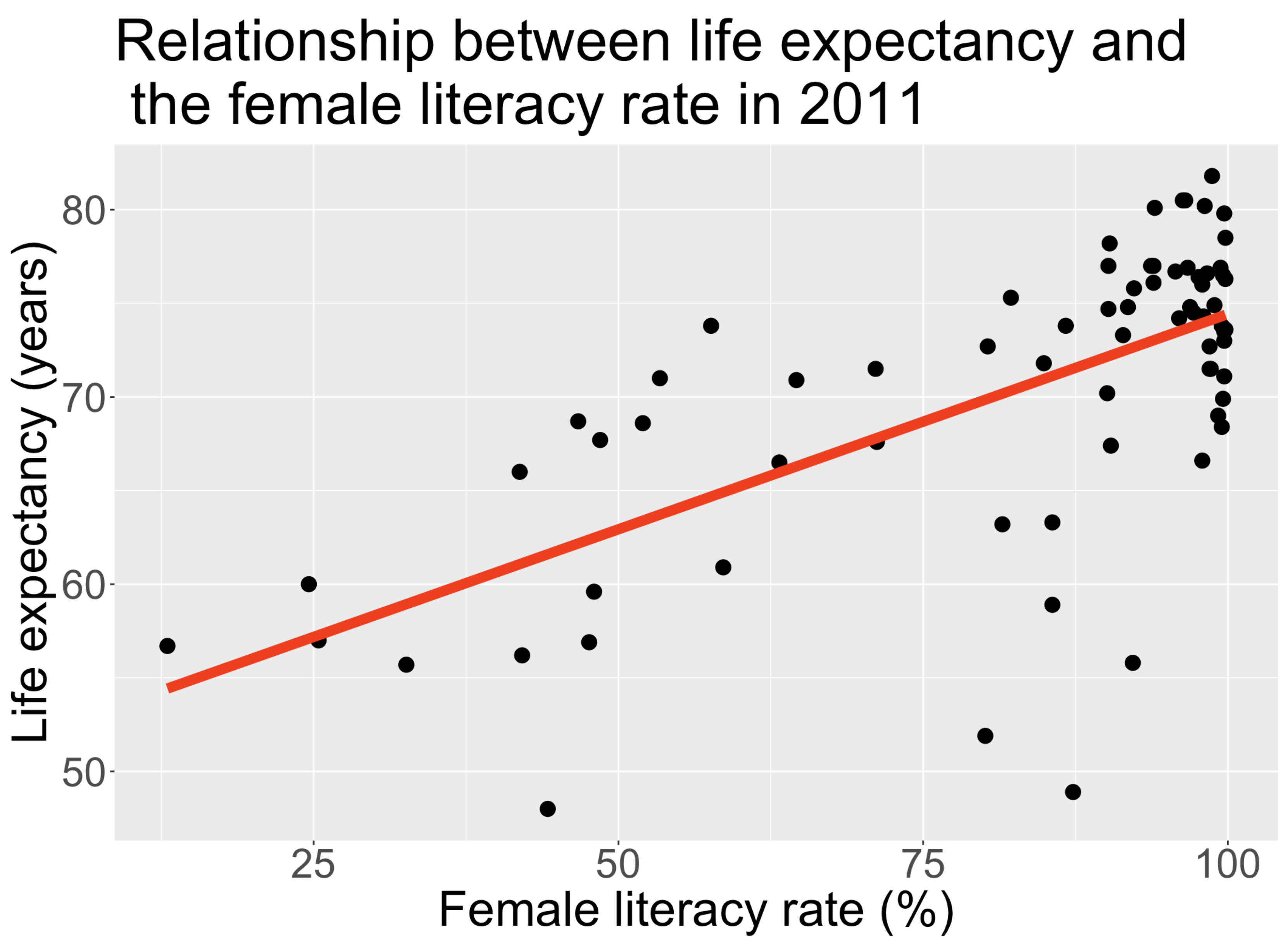
Step 1: Simple linear regressions / analysis

- For each covariate, we want to see how it relates to the outcome (without adjusting for other covariates)
- We can partially do this with **visualizations**
  - Helps us see the data we throw it into regression that makes assumptions (like our LINE assumptions)
  - `ggpairs()` can be a quick way to do it
  - `ggplot()` can make each plot
    - `+ geom_boxplot()` to make boxplots by groups for categorical covariates
    - `+ geom_jitter() + stat_summary()` to make non-overlaping points with group means for categorical covariates
    - `+ geom_point()` to make scatterplots for continuous covariates
- We need to run **simple linear regression**
  - We're calling regression with multi-level categories "simple" even though there are multiple coefficients

Step 1: Simple linear regressions / analysis

- Let's think back to our Gapminder dataset
- Always good to start with our main relationship: life expectancy vs. female literacy rate
  - *Throwback to Lesson 3 SLR when we first visualized and ran `lm()` for this relationship*

```
model_FLR = lm(LifeExpectancyYrs ~ FemaleLiteracyRate, data = gapm_sub)
```



term	estimate	std.error	statistic	p.value
(Intercept)	51.438	2.739	18.782	0.000
FemaleLiteracyRate	0.230	0.032	7.141	0.000

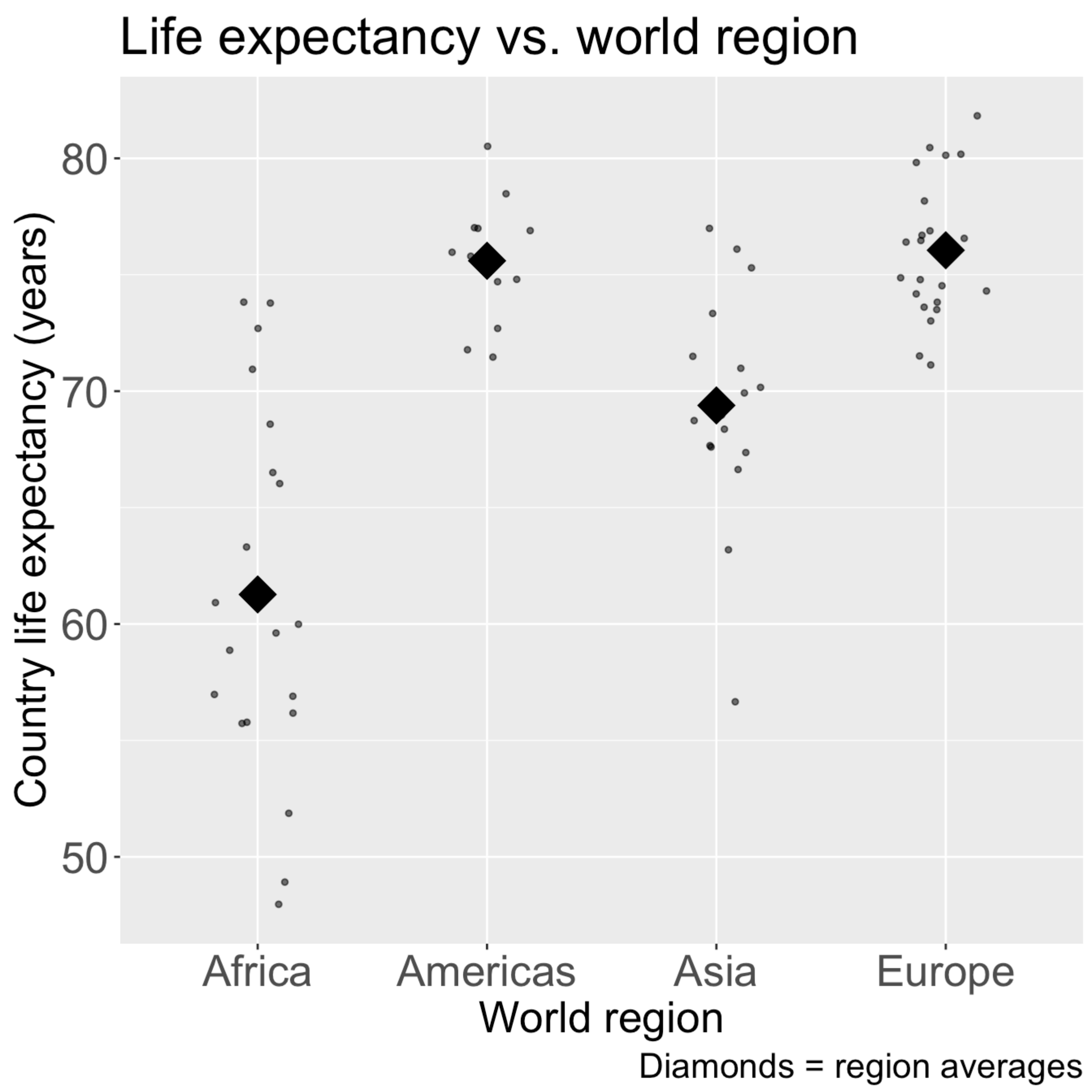
Poll Everywhere Question 4

Step 1: Simple linear regressions / analysis

- Let’s do this with one other variable before I show you a streamlined version of SLR

```
model_WR = lm(LifeExpectancyYrs ~ four_regions, data = gapm_sub)
```

▶ Code




```
anova(model_WR) %>% tidy() %>% gt() %>%
  tab_options(table.font.size = 40) %>%
  fmt_number(decimals = 3)
```

term	df	sumsq	meansq	statistic	p.value
four_regions	3.000	2,743.042	914.347	33.680	0.000
Residuals	68.000	1,846.077	27.148	NA	NA

- Recall from Lesson 5 (SLR: More inference + Evaluation):
  - `anova()` with one model name will compare the model (`model_WR`) to the intercept model

Step 1: Simple linear regressions / analysis

- If we do a good job visualizing the relationship between our outcome and each covariate, then we can proceed to a streamlined version of the F-test for each relationship
- Run `add1()` to add each variable one at a time and separately
- Output will include hypothesis test (using F-test) if coefficient(s) is 0 or not

```r
intercept_model = gapm2 %>% lm(formula = LifeExpectancyYrs ~ 1)
add1(intercept_model, scope = ~ FemaleLiteracyRate + CO2emissions + IncomePP + four_regions +
       WaterSourcePrct + FoodSupplykcPPD + members_oecd_g77, test = "F")
```

```
Single term additions

Model:
LifeExpectancyYrs ~ 1
                   Df Sum of Sq    RSS    AIC  F value    Pr(>F)
<none>                          4589.1 301.14
FemaleLiteracyRate  1   1934.24 2654.9 263.74  50.9994 6.895e-10 ***
CO2emissions        1    452.31 4136.8 295.67   7.6536  0.007241 **
IncomePP            1   1220.34 3368.8 280.89  25.3576 3.557e-06 ***
four_regions        3   2743.04 1846.1 241.58  33.6799 1.858e-13 ***
WaterSourcePrct     1   2988.20 1600.9 227.32 130.6592 < 2.2e-16 ***
FoodSupplykcPPD     1   1893.44 2695.7 264.84  49.1679 1.188e-09 ***
members_oecd_g77    2   1103.71 3485.4 285.34  10.9250 7.553e-05 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Step 2: Preliminary variable selection

- Identify candidates for your first multivariable model by performing an F-test on each covariate's SLR
  - Using p-values from previous slide
  - If the p-value of the test is less than 0.25, then consider the variable a candidate

- Candidates for first multivariable model
  - All clinically important variables (regardless of p-value)
  - Variables with univariate test with p-value < 0.25

- With more experience, you won't need to rely on these strict rules as much

Step 2: Preliminary variable selection

- From the previous p-values from the F-test on each covariate's SLR
  - Decision: we keep all the covariates since they all have a p-value < 0.25

```r
add1(intercept_model, scope = ~ FemaleLiteracyRate + CO2emissions + IncomePP + four_regions +
       WaterSourcePrct + FoodSupplykcPPD + members_oecd_g77, test = "F")
```

```
Single term additions

Model:
LifeExpectancyYrs ~ 1
                   Df Sum of Sq    RSS    AIC  F value    Pr(>F)    
<none>                          4589.1 301.14                        
FemaleLiteracyRate  1   1934.24 2654.9 263.74  50.9994 6.895e-10 ***
CO2emissions        1    452.31 4136.8 295.67   7.6536  0.007241 ** 
IncomePP            1   1220.34 3368.8 280.89  25.3576 3.557e-06 ***
four_regions        3   2743.04 1846.1 241.58  33.6799 1.858e-13 ***
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members_oecd_g77    2   1103.71 3485.4 285.34  10.9250 7.553e-05 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Step 2: Preliminary variable selection

- Fit an **initial model** including any independent variable with p-value < 0.25 and clinically important variables

```r
init_model = lm(LifeExpectancyYrs ~
                  FemaleLiteracyRate +
                  CO2emissions +
                  IncomePP +
                  four_regions +
                  WaterSourcePrct +
                  FoodSupplykcPPD +
                  members_oecd_g77,
                data = gapm2)

tbl_regression(
  init_model,
  label = list(
    FemaleLiteracyRate ~ "Female literacy rate (%)",
    CO2emissions ~ "CO2 emissions",
    IncomePP ~ "Income  (GDP per capita)",
    four_regions ~ "World region",
    WaterSourcePrct ~ "Access to omproved water (%)",
    FoodSupplykcPPD ~ "Food supply (kcal PPD)",
    members_oecd_g77 ~ "Intergovernmental group"
    )) %>%
  as_gt() %>%
  tab_options(table.font.size = 26)
```

Characteristic	**Beta**	**95% CI**[1]	**p-value**
Female literacy rate (%)	0.00	-0.07, 0.07	>0.9
CO2 emissions	-0.29	-0.55, -0.02	0.037
Income (GDP per capita)	0.00	0.00, 0.00	0.019
World region			
Africa	—	—	
Americas	9.9	5.9, 14	<0.001
Asia	5.8	2.6, 9.0	<0.001
Europe	7.1	1.7, 13	0.010
Access to omproved water (%)	0.14	0.01, 0.27	0.041
Food supply (kcal PPD)	0.01	0.00, 0.01	0.015
Intergovernmental group			
g77	—	—	
oecd	-0.33	-5.4, 4.8	0.9
others	0.33	-4.3, 4.9	0.9

[1] CI = Confidence Interval

Step 3: Assess change in coefficient

- This is where we start identifying covariates that we might remove
- I would start by using the p-value to guide me towards specific variables
  - Female literacy rate, but that's our main covariate
  - Intergovernmental group?
  - Maybe water source percent?
- Some people will say you can use the p-value alone
  - I like to double check that those variables do not have a large effect on the other coefficients

Characteristic	Beta	95% CI[1]	p-value
Female literacy rate (%)	0.00	-0.07, 0.07	>0.9
CO2 emissions	-0.29	-0.55, -0.02	0.037
Income (GDP per capita)	0.00	0.00, 0.00	0.019
World region			
Africa	—	—	
Americas	9.9	5.9, 14	<0.001
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Europe	7.1	1.7, 13	0.010
Access to omproved water (%)	0.14	0.01, 0.27	0.041
Food supply (kcal PPD)	0.01	0.00, 0.01	0.015
Intergovernmental group			
g77	—	—	
oecd	-0.33	-5.4, 4.8	0.9
others	0.33	-4.3, 4.9	0.9

[1] CI = Confidence Interval

Step 3: Assess change in coefficient

- Very similar to the process we used when looking at confounders

- One variable at a time, we run the multivariable model with and without a variable
  - We look at the p-value of the F-test for the coefficients of said variable
  - We look at the percent change for the coefficient ($\Delta\%$) of our **explanatory variable** (FLR in our example)

- General rule: We can remove a variable if…
  - p-value > 0.05 for the F-test of its own coefficients
  - AND change in coefficient ($\Delta\%$) of our explanatory variable is < 10%

F-test on dropping each covariate

- Function `drop1()`: If we put in our initial model, the function will remove each covariate and perform the respective F-test to test if the coefficients are 0 (null) or not (alternative).

```
drop1(init_model, test="F")
```

```
Single term deletions

Model:
LifeExpectancyYrs ~ FemaleLiteracyRate + CO2emissions + IncomePP +
    four_regions + WaterSourcePrct + FoodSupplykcPPD + members_oecd_g77
                   Df Sum of Sq     RSS    AIC F value    Pr(>F)
<none>                           999.20 211.38
FemaleLiteracyRate  1      0.06  999.26 209.38  0.0034   0.95391
CO2emissions        1     74.63 1073.83 214.57  4.5559   0.03683 *
IncomePP            1     95.40 1094.60 215.95  5.8240   0.01883 *
four_regions        3    410.14 1409.34 230.14  8.3462 9.822e-05 ***
WaterSourcePrct     1     71.74 1070.94 214.37  4.3799   0.04053 *
FoodSupplykcPPD     1    102.06 1101.26 216.38  6.2305   0.01528 *
members_oecd_g77    2      1.79 1000.99 207.51  0.0546   0.94696
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Testing for percent change ($\Delta\%$) in a coefficient

- Let's say we have X_1 and X_2, and we specifically want to see if X_2 is a confounder for X_1 (the explanatory variable or variable of interest)
- If we are only considering X_1 and X_2, then we need to run the following two models:
  - Fitted model 1 / reduced model (`mod1`): $\widehat{Y} = \widehat{\beta}_0 + \widehat{\beta}_1 X_1$
    - We call the above $\widehat{\beta}_1$ the reduced model coefficient: $\widehat{\beta}_{1,\text{mod1}}$ or $\widehat{\beta}_{1,\text{red}}$
  - Fitted model 2 / Full model (`mod2`): $\widehat{Y} = \widehat{\beta}_0 + \widehat{\beta}_1 X_1 + \widehat{\beta}_2 X_2$
    - We call this $\widehat{\beta}_1$ the full model coefficient: $\widehat{\beta}_{1,\text{mod2}}$ or $\widehat{\beta}_{1,\text{full}}$

Calculation for % change in coefficient

$$\Delta\% = 100\% \cdot \frac{\widehat{\beta}_{1,\text{mod1}} - \widehat{\beta}_{1,\text{mod2}}}{\widehat{\beta}_{1,\text{mod2}}} = 100\% \cdot \frac{\widehat{\beta}_{1,\text{red}} - \widehat{\beta}_{1,\text{full}}}{\widehat{\beta}_{1,\text{full}}}$$

Step 3: Assess change in coefficient

- Let's try this out on `members_oecd_g77`

▶ Display the ANOVA table with F-statistic and p-value

term	df.residual	rss	df	sumsq	statistic	p.value
LifeExpectancyYrs ~ FemaleLiteracyRate + CO2emissions + IncomePP + four_regions + WaterSourcePrct + FoodSupplykcPPD + members_oecd_g77	61.000	999.201	NA	NA	NA	NA
LifeExpectancyYrs ~ FemaleLiteracyRate + CO2emissions + IncomePP + four_regions + WaterSourcePrct + FoodSupplykcPPD	63.000	1,000.988	−2.000	−1.787	0.055	0.947

- $\widehat{\beta}_{FLR,full} = 0.002, \widehat{\beta}_{FLR,red} = 0.0036$

$$\Delta\% = 100\% \cdot \frac{\widehat{\beta}_{FLR,full} - \widehat{\beta}_{FLR,red}}{\widehat{\beta}_{FLR,full}} = 100\% \cdot \frac{0.002 - 0.0036}{0.002} = -74.41\%$$

- Based off the percent change, I would keep this in the model

Step 3: Assess change in coefficient (Reference only)

- Let's try this out on water source percent (even though the p-value was < 0.05)

▶ Display the ANOVA table with F-statistic and p-value

term	df.residual	rss	df	sumsq	statistic	p.value
LifeExpectancyYrs ~ FemaleLiteracyRate + CO2emissions + IncomePP + four_regions + WaterSourcePrct + FoodSupplykcPPD + members_oecd_g77	61.000	999.201	NA	NA	NA	NA
LifeExpectancyYrs ~ FemaleLiteracyRate + CO2emissions + IncomePP + four_regions + members_oecd_g77 + FoodSupplykcPPD	62.000	1,070.944	−1.000	−71.744	4.380	0.041

- $\widehat{\beta}_{FLR,full} = 0.002, \widehat{\beta}_{FLR,red} = 0.034$

$$\Delta\% = 100\% \cdot \frac{\widehat{\beta}_{FLR,full} - \widehat{\beta}_{FLR,red}}{\widehat{\beta}_{FLR,full}} = 100\% \cdot \frac{0.002 - 0.034}{0.002} = -1561.06\%$$

- Based off the percent change (and p-value), I would keep this in the model

Poll Everywhere Question 5

Step 3: Assess change in coefficient: Summary

- At the end of this step, we have a **preliminary main effects model**
- Where the variables are excluded that met the following criteria:
  - P-value > 0.05 for the F-test of its own coefficients
  - Change in coefficient ($\Delta\%$) of our explanatory variable is < 10%
- In our example, the **preliminary main effects model** (end of Step 3) was the same as the **initial model** (end of Step 2)
- Preliminary main effects model includes:
  - `FemaleLiteracyRate`
  - `CO2emissions`
  - `IncomePP`
  - `four_regions`
  - `members_oecd_g77`
  - `FoodSupplykcPPD`
  - `WaterSupplePct`

Recap of Steps 1-3

- Pre-step: Exploratory data analysis
- Step 1: Simple linear regressions / analysis
  - Look at each covariate with outcome
  - Perform SLR for each covariate
- Step 2: Preliminary variable selection
  - From SLR, decide which variables go into the initial model
  - Use F-test to see if each covariate (on its own) explains enough variation in outcome
  - End with **initial model**
- Step 3: Assess change in coefficients
  - From the initial model at end of step 2, we take a variable out of the model if:
    - P-value > 0.05 for the F-test of its own coefficients
    - Change in coefficient ($\Delta\%$) of our explanatory variable is < 10%
  - End with **preliminary main effects model**

Learning Objectives

1. Understand the overall steps for purposeful selection as a model building strategy

2. Apply purposeful selection to a dataset using R

3. Use different approaches to assess the linear scale of continuous variables in logistic regression

Step 4: Assess scale for continuous variables

- We assume the linear regression model is linear for **each continuous variable**
- We need to assess linearity for continuous variables in the model
  - Do this through smoothed scatterplots that we introduced in Lesson 6 (SLR Diagnostics)
  - Residual plots (can be used in SLR) does not help us in MLR
  - Each term in MLR model needs to have linearity with outcome
- Three methods/approaches to address the violation of linearity assumption:
  - **Approach 1:** Categorize continuous variable
  - **Approach 2:** Fractional Polynomials
  - **Approach 3:** Spline functions
- Approach will depend on the covariate!!
- For our class, only implement **Approach 1 or 2**
- Model at the end of Step 4 is the **main effects model**

Step 4: Assess scale for continuous variables: Smoothed scatterplots

- Smoother scatterplots **only check linearity**, not addressing linearity issues

- Can also identify extreme observations
  - Again, just want to flag these values
  - Can influence the assessment of linearity when using fractional polynomials or spline functions

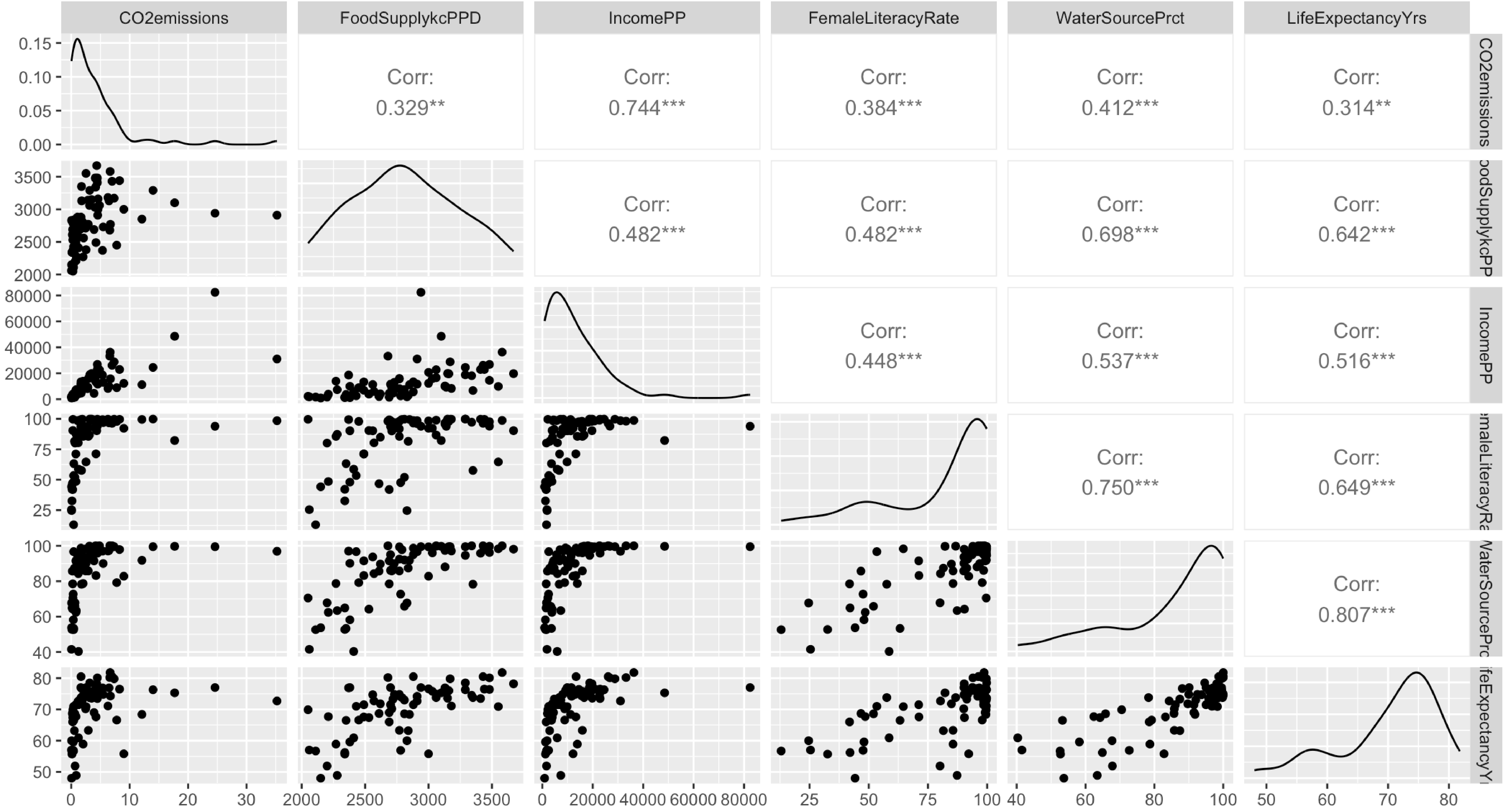
- Helps us decide if the continuous variable can stay **as is** in the model
  - **Problem:** if not linear, then we need to represent the variable in a new way (Approaches 1-3)

Step 4: Assess scale for continuous variables: Smoothed scatterplots

- In Gapminder dataset, we have 5 continuous variables:
  - CO2 Emissions
  - Food Supply
  - Income
  - Female Literacy Rate
  - Water source percent
- Plot each of these agains the outcome, life expectancy

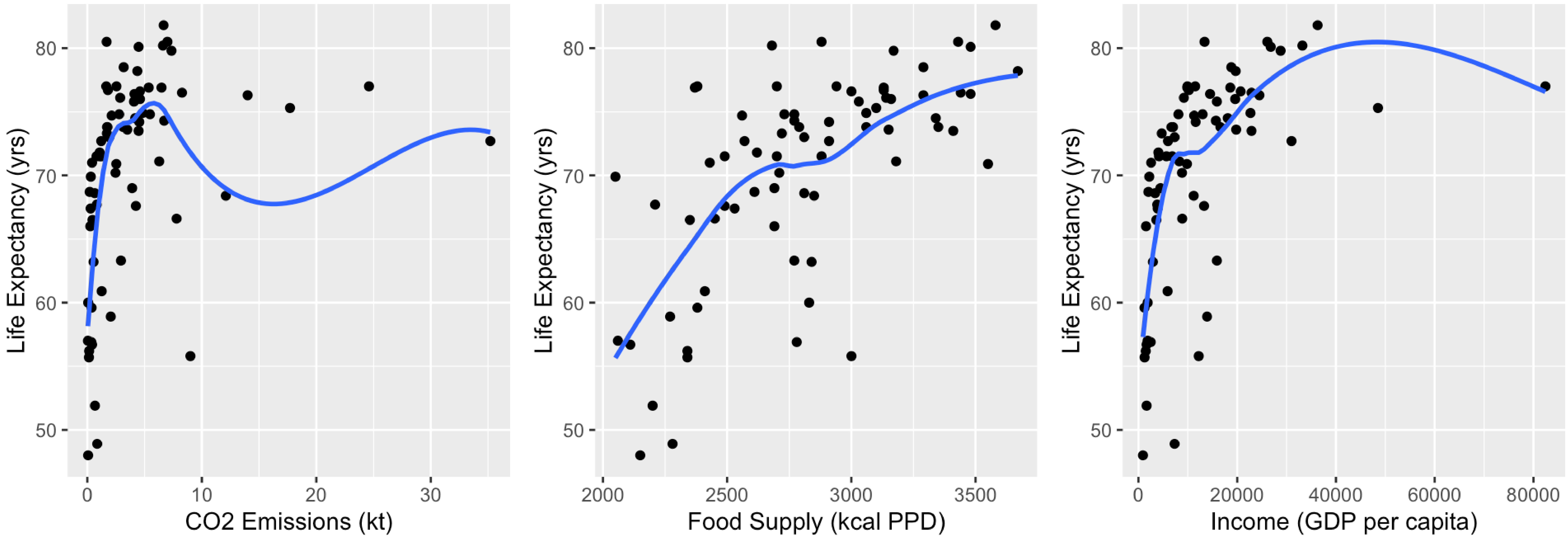
Step 4: Assess scale for continuous variables: Smoothed scatterplots

- We can quickly look at ggpairs() to identify variables



Step 4: Assess scale for continuous variables: Smoothed scatterplots

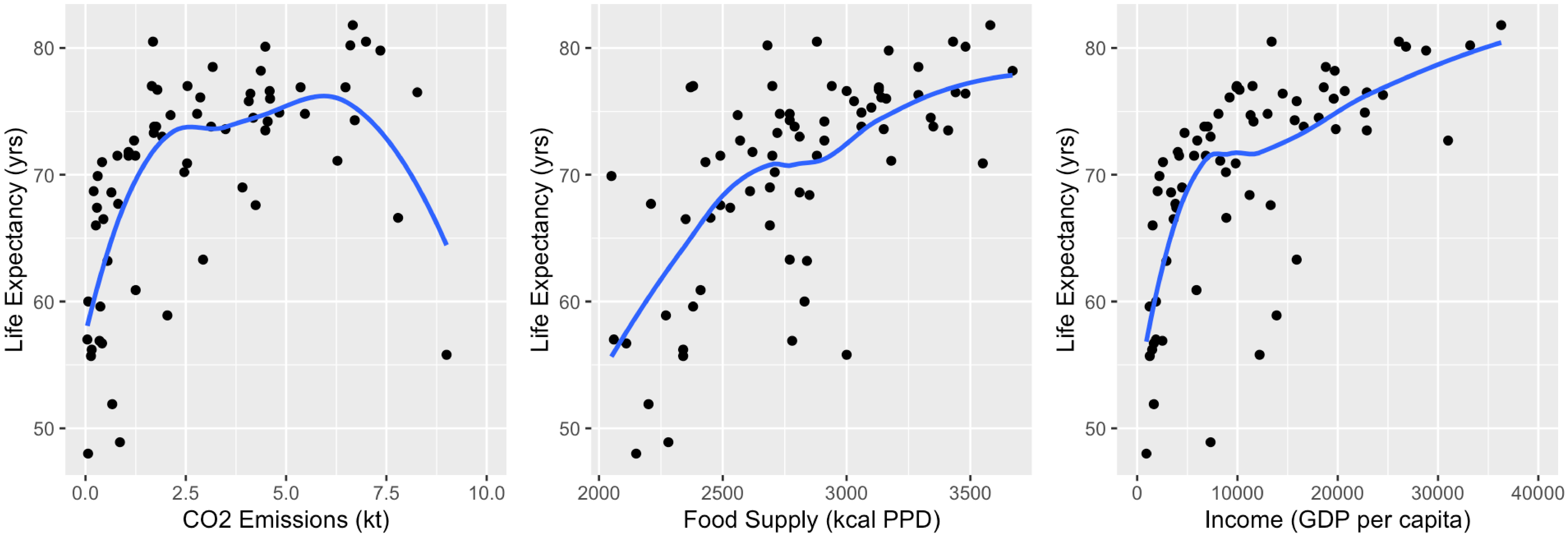
- Take a look at C02, Food Supply, and Income



- Food Supply looks admissible
- CO2 Emissions and Income do not look very linear, but I want to zoom into the area of the plots that have most of the data

Step 4: Assess scale for continuous variables: Smoothed scatterplots

- Zoom into areas on plots with more data



- Food Supply still looks admissible
- CO2 Emissions and Income not linear: will address this!!

Step 4: Assess scale for continuous variables

- Three methods/approaches to address the violation of linearity assumption:
  - **Approach 1:** Categorize continuous variable
  - **Approach 2:** Fractional Polynomials
  - **Approach 3:** Spline functions

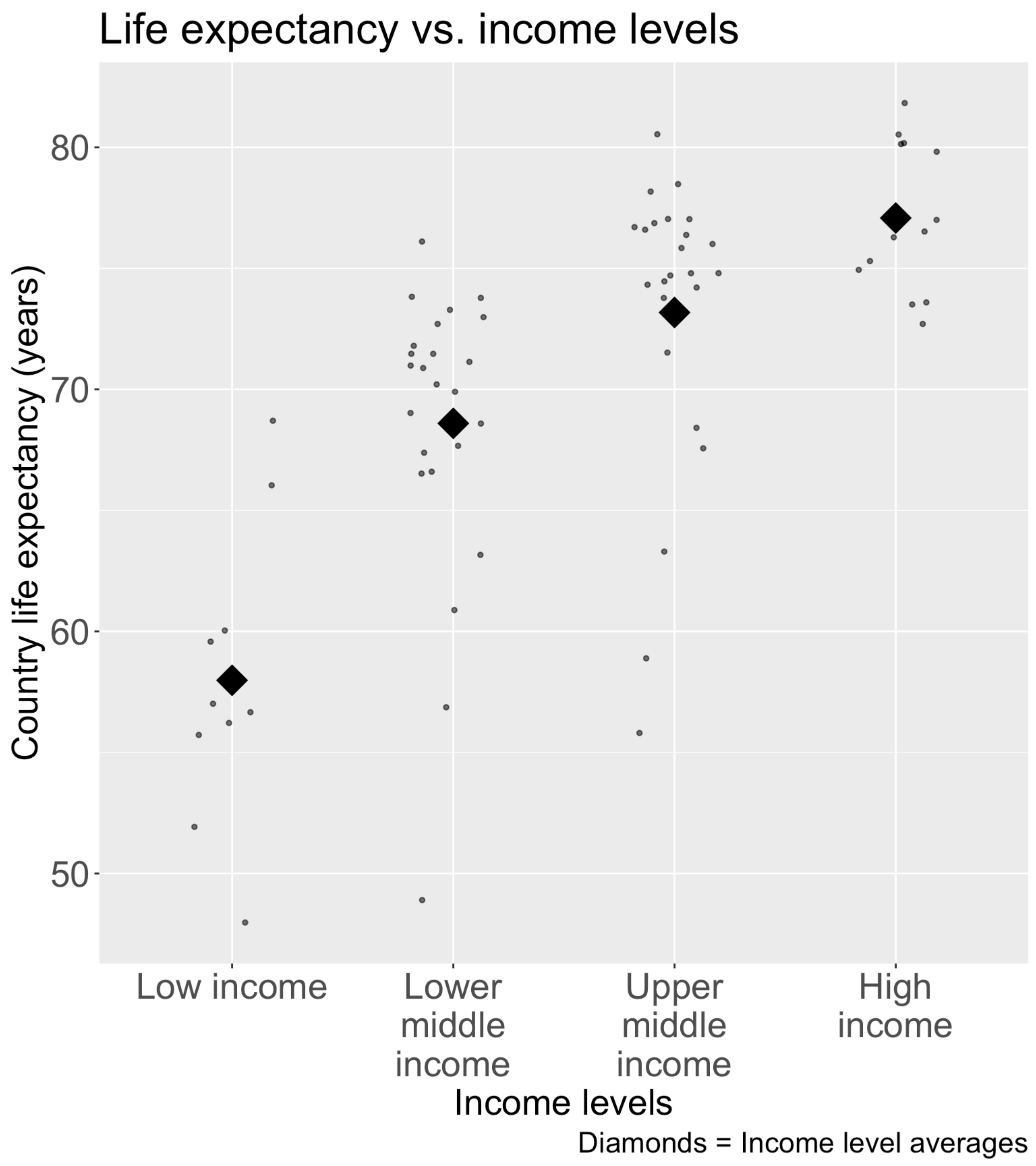
Step 4: Approach 1: Categorize continuous variable

- Categorize continuous variables
  - Percentiles, quartiles, quantiles
    - Create indicator variables corresponding to each quartile
  - Meaningful thresholds
    - Example: income level groups discussed by Gapminder
- Disadvantages:
  - Takes some time to create new variables, especially with multiple continuous covariates
  - Start with quartiles, but might be more appropriate to use different splits
    - No set rules on this
- Advantage: graphical and visually helps

Step 4: Approach 1: Categorize continuous variable

- For income, I would use Gapminder's income level groups
  - Discussed in Lesson 10 Categorical Covariates (slide 43)

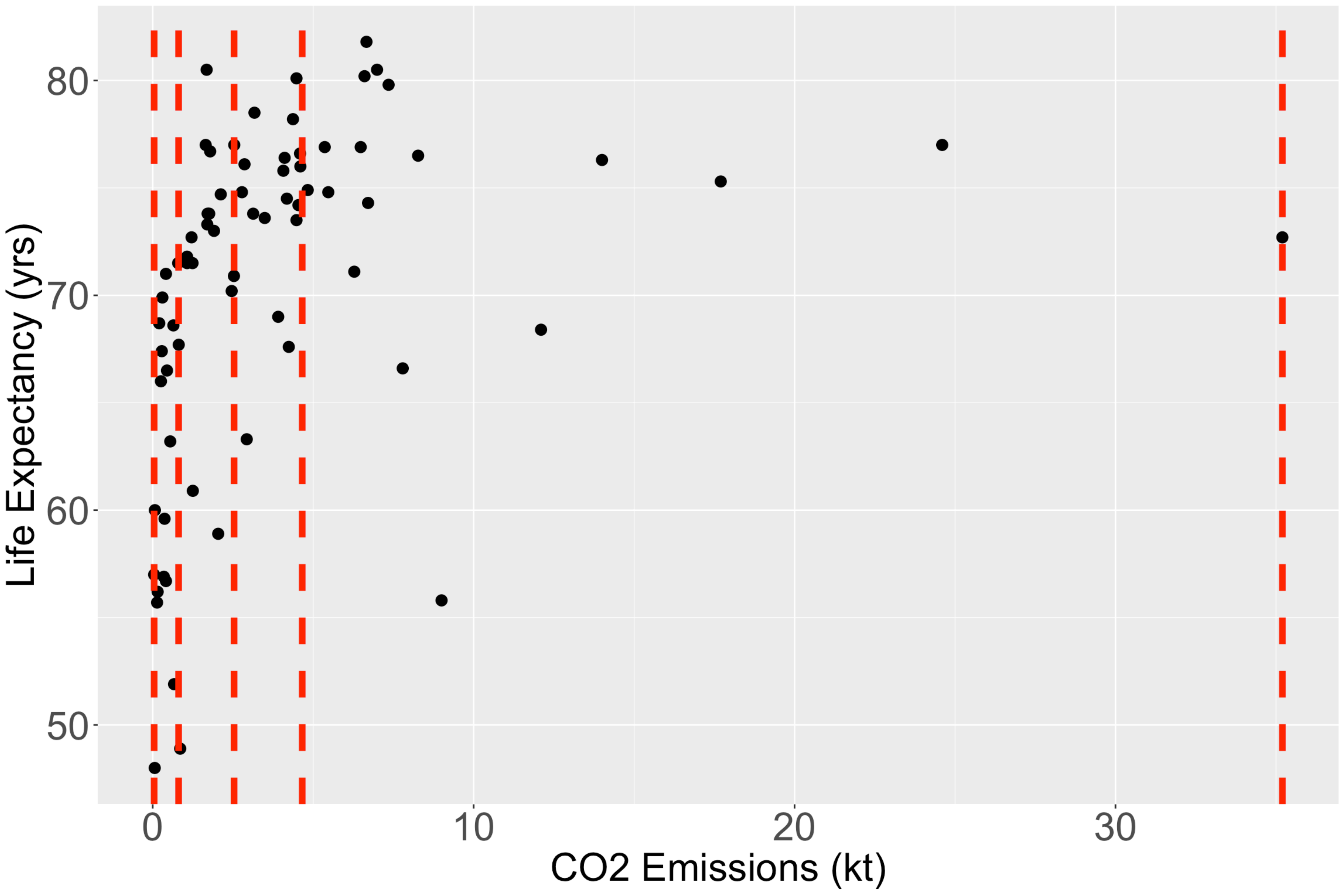
- Experts in the field have developed these income groups
  - I think this is best solution for income (that was not meeting linearity as a continuous variable)



Step 4: Approach 1: Categorize continuous variable

- Let's still try it out with CO2 Emissions (kt)
- I have plotted the quartile lines of food supply with red lines

▶ Take a look at the quartiles within the scatterplot

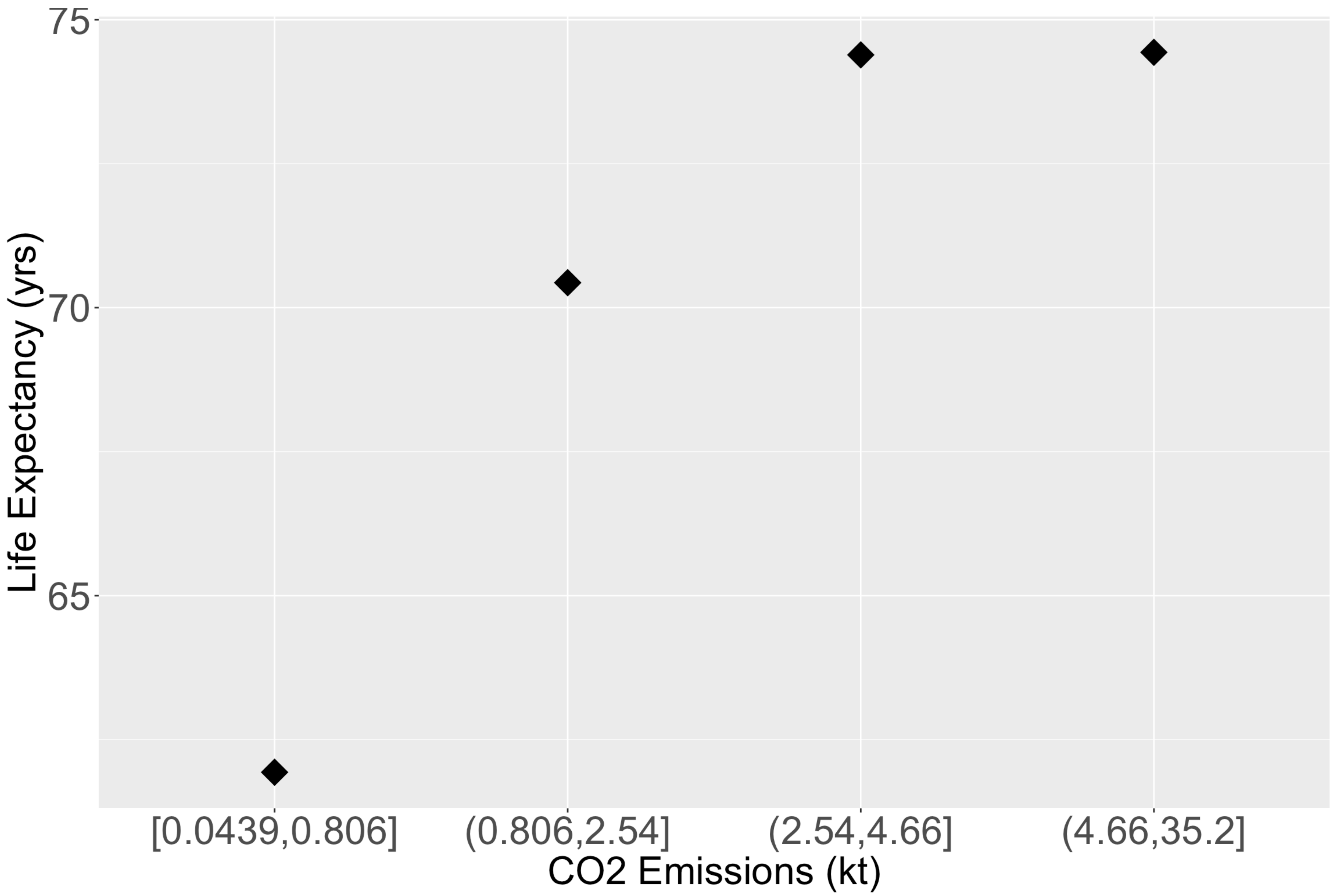


Step 4: Approach 1: Categorize continuous variable

- Let's make the quartiles for CO2 emissions:

```
library(dvmisc)
gapm2 = gapm2 %>%
  mutate(CO2_q = quant_groups(CO2emissions, groups = 4) %>% factor())
```

- Take a look at the quartile means within the scatterplot



Step 4: Approach 1: Categorize continuous variable

- Let's fit a new model with the two new representations for income and CO2 emissions

- Remember, this is the **main effects model** if we decide to make CO2 into quartiles

term	estimate	std.error	statistic	p.value
(Intercept)	39.877	4.889	8.157	0.000
FemaleLiteracyRate	−0.073	0.047	−1.555	0.125
CO2_q(0.806,2.54]	1.099	1.914	0.574	0.568
CO2_q(2.54,4.66]	−0.292	2.419	−0.121	0.904
CO2_q(4.66,35.2]	−0.595	2.524	−0.236	0.814
income_levels1Lower middle income	5.441	2.343	2.322	0.024
income_levels1Upper middle income	6.111	2.954	2.069	0.043
income_levels1High income	7.959	3.277	2.429	0.018
four_regionsAmericas	9.003	2.050	4.391	0.000
four_regionsAsia	5.260	1.637	3.213	0.002
four_regionsEurope	6.855	2.871	2.387	0.020
WaterSourcePrct	0.166	0.066	2.496	0.015
FoodSupplykcPPD	0.004	0.002	1.825	0.073
members_oecd_g77oecd	1.119	2.674	0.418	0.677
members_oecd_g77others	1.047	2.511	0.417	0.678

Step 4: Approach 2: Fractional Polynomials

- Main concepts and transformations presented in Lesson 7 SLR: Model Evaluation and Diagnostics (slide 33 on)
- Idea: test many transformations of a continuous covariate
  - Based on Royston and Altman, Applied Statistics, 1994

- Recall Tukey's transformation (power) ladder
  - And can use R's `gladder()` to see the transformations

Power p	-3	-2	-1	-1/2	0	1/2	1	2	3
	$\frac{1}{x^3}$	$\frac{1}{x^2}$	$\frac{1}{x}$	$\frac{1}{\sqrt{x}}$	$\log(x)$	$\sqrt{x}$	x	x^2	x^3

- We can run through each and test different models, or use the approach from Lesson 7
- There is also a package we can use!
  - mfp package in R contains the fp() function

Step 4: Approach 2: Fractional Polynomials

```r
library(mfp)

fp_model_CO2 = mfp(LifeExpectancyYrs ~ FemaleLiteracyRate +
                     fp(CO2emissions, df = 4) + income_levels1 + four_regions +
                     WaterSourcePrct + FoodSupplykcPPD + members_oecd_g77,
                 data = gapm2, family = "gaussian")

fp_model_CO2$fptable %>% gt(rownames_to_stub = T) %>% tab_options(table.font.size = 24)
```

	df.initial	select	alpha	df.final	power1	power2
four_regionsAmericas	1	1	0.05	1	1	.
four_regionsAsia	1	1	0.05	1	1	.
four_regionsEurope	1	1	0.05	1	1	.
WaterSourcePrct	1	1	0.05	1	1	.
income_levels1Lower middle income	1	1	0.05	1	1	.
income_levels1Upper middle income	1	1	0.05	1	1	.
income_levels1High income	1	1	0.05	1	1	.
FoodSupplykcPPD	1	1	0.05	1	1	.
FemaleLiteracyRate	1	1	0.05	1	1	.
CO2emissions	4	1	0.05	1	1	.
members_oecd_g77oecd	1	1	0.05	1	1	.
members_oecd_g77others	1	1	0.05	1	1	.

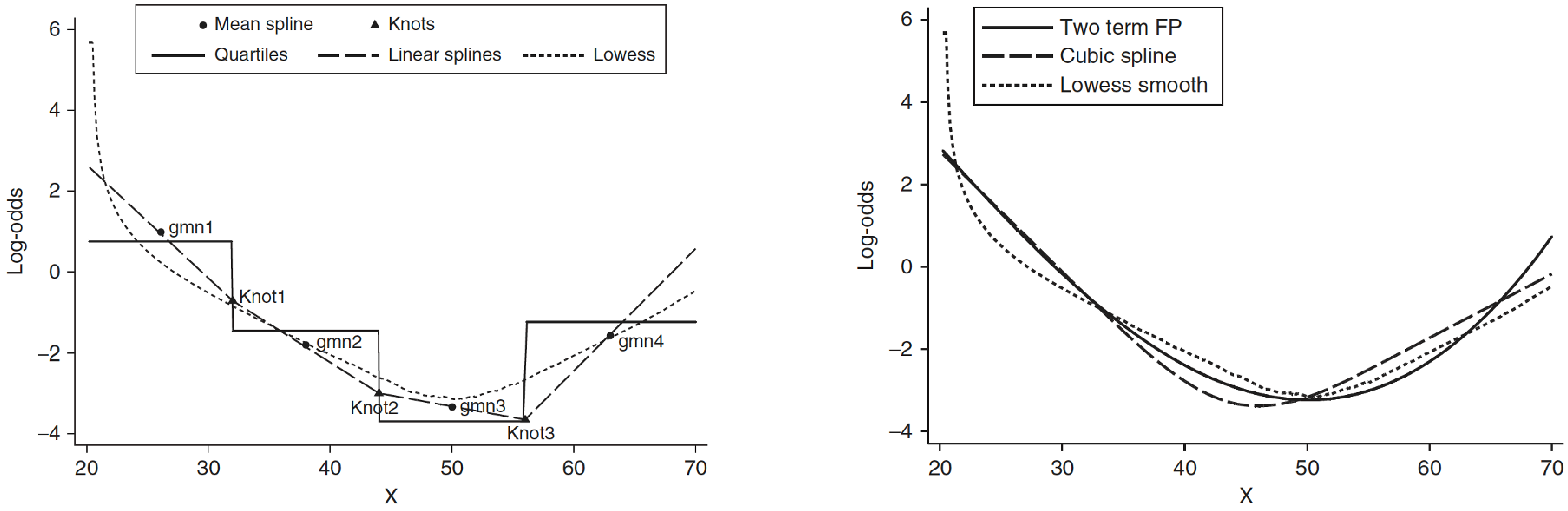
Step 4: Approach 2: Fractional Polynomials

	df.initial	select	alpha	df.final	power1	power2
four_regionsAmericas	1	1	0.05	1	1	.
four_regionsAsia	1	1	0.05	1	1	.
four_regionsEurope	1	1	0.05	1	1	.
WaterSourcePrct	1	1	0.05	1	1	.
income_levels1Lower middle income	1	1	0.05	1	1	.
income_levels1Upper middle income	1	1	0.05	1	1	.
income_levels1High income	1	1	0.05	1	1	.
FoodSupplykcPPD	1	1	0.05	1	1	.
FemaleLiteracyRate	1	1	0.05	1	1	.
CO2emissions	4	1	0.05	1	1	.
members_oecd_g77oecd	1	1	0.05	1	1	.
members_oecd_g77others	1	1	0.05	1	1	.

- Conclusion from fractional polynomial is that CO2 does not need to be transformed
- A little counter-intuitive to what we saw in quartiles
- Thus, I think leaving CO2 emissions as quartiles is best!

Step 4: Approach 3: Spline functions

- Spline function is to fit a series of smooth curves that joined at specific points (called knots)



Step 4: Approach 3: Spline functions

- Need to specify knots for spline functions
  - More knots are flexible, but requires more parameters to estimate
  - In most applications three to five knots are sufficient

- Within our class, fractional polynomials will be sufficient

- If you think this is cool, I highly suggest you look into Functional Data Analysis (FDA) or Functional Regression
  - Jeffrey Morris is a big name in that field

- In R there are a few options to incorporate splines
  - `pspline( )`: More information
  - `smoothHR()`: More information

Step 4 Conclusion: main effects model

- We concluded that we will use:
  - Income levels (categorical) that Gapminder created
  - Quartiles for CO_2 Emissions

> **Note**
>
> This is also a good step to decide if you would like to score a categorical variable (Lesson 5)

- Question: Do you see any visual issues with my regression table?

Characteristic	Beta	95% CI[1]	p-value
Female literacy rate (%)	-0.07	-0.17, 0.02	0.13
CO2 emissions quartiles			
[0.0439,0.806]	—	—	
(0.806,2.54]	1.1	-2.7, 4.9	0.6
(2.54,4.66]	-0.29	-5.1, 4.6	>0.9
(4.66,35.2]	-0.60	-5.6, 4.5	0.8
Income levels			
Low income	—	—	
Lower middle income	5.4	0.75, 10	0.024
Upper middle income	6.1	0.20, 12	0.043
High income	8.0	1.4, 15	0.018
World region			
Africa	—	—	
Americas	9.0	4.9, 13	<0.001
Asia	5.3	2.0, 8.5	0.002
Europe	6.9	1.1, 13	0.020
Access to omproved water (%)	0.17	0.03, 0.30	0.015
Food supply (kcal PPD)	0.00	0.00, 0.01	0.073
Intergovernmental group			
g77	—	—	
oecd	1.1	-4.2, 6.5	0.7
others	1.0	-4.0, 6.1	0.7

[1] CI = Confidence Interval

Learning Objectives

1. Understand the overall steps for purposeful selection as a model building strategy

2. Apply purposeful selection to a dataset using R

3. Use different approaches to assess the linear scale of continuous variables in logistic regression

Step 5: Check for interactions

- Create a list of interaction terms from variables in the "main effects model" that has clinical plausibility

- Add the interaction variables, one at a time, to the main effects model, and assess the significance using a F-test
  - May keep interaction terms with p-value < 0.10 (or 0.05)

- Keep the main effects untouched, only simplify the interaction terms

- Use methods from Step 2 (comparing model with all interactions to a smaller model with interactions) to determine which interactions to keep

- The model by the end of Step 5 is called the **preliminary final model**

Step 5: Check for interactions

- We test with $\alpha = 0.10$
- Follow the F-test procedure in Lesson 10 (MLR: Using the F-test)
  - This means we need to follow the 7 steps of the general F-test in previous slide (taken from Lesson 10)
- Use the hypothesis tests for the specific variable combo:

Binary & continuous variable (Lesson 11, LOB 2)

Testing a single coefficient for the interaction term using F-test comparing full model to reduced model

Multi-level & continuous variables (Lesson 11, LOB 3)

Testing group of coefficients for the interaction terms using F-test comparing full to reduced model

Binary & multi-level variable (Lesson 12, LOB 4)

Testing group of coefficients for the interaction terms using F-test comparing full to reduced model

Two continuous variables (Lesson 12, LOB 5)

Testing a single coefficient for the interaction term using F-test comparing full to reduced model

Poll Everywhere Questions 6-8

Step 5: Check for interactions

- Use `add1()` function to compare a full model (interactions with FLR) and reduced model (main effects model)

```
add1(main_eff_model, scope = ~ FemaleLiteracyRate * . ,  test = "F")
```

```
Single term additions

Model:
LifeExpectancyYrs ~ FemaleLiteracyRate + CO2_q + income_levels1 +
    four_regions + WaterSourcePrct + FoodSupplykcPPD + members_oecd_g77
                                   Df Sum of Sq    RSS    AIC F value Pr(>F)
<none>                                          946.46 215.48
FemaleLiteracyRate:CO2_q            3    27.171 919.29 219.38  0.5320 0.6623
FemaleLiteracyRate:income_levels1   3    12.802 933.66 220.50  0.2468 0.8633
FemaleLiteracyRate:four_regions     3    95.987 850.47 213.78  2.0315 0.1203
FemaleLiteracyRate:WaterSourcePrct  1     3.040 943.42 217.25  0.1804 0.6726
FemaleLiteracyRate:FoodSupplykcPPD  1    31.063 915.40 215.07  1.9003 0.1735
FemaleLiteracyRate:members_oecd_g77 2    11.513 934.95 218.59  0.3386 0.7142
```

- I went through all the ANOVA tables, and found the following significant interactions:
  - None!
- **Think about it:** does that track with what we saw in our interactions lecture?

Step 6: Assess model fit

- Assess the adequacy of the model (diagnostics) and check its fit

- Methods for diagnostics will be discussed next class
  - Combination of diagnostics and model fit statistics!
  - Looked at model fit statistics in last lesson
  - Look at diagnostics in Lesson 15: MLR Diagnostics

- If the model is adequate and fits well, then it is the **Final model**

Step 6: Assess model fit

- Our **final model** contains
  - Female Literacy Rate `FLR`
  - CO2 Emissions in quartiles `CO2_q`
  - Income levels in groups assigned by Gapminder `income_levels`
  - World regions `four_regions`
  - Membership of global and economic groups `members_oecd_g77`
    - OECD: Organization for Economic Co-operation and Development
    - G77: Group of 77
    - Other
  - Food Supply `FoodSupplykcPPD`
  - Clean Water Supply `WaterSupplePct`

Step 6: Assess model fit: Model fit statistics

- Way I did it in the lab instructions (and last class)

```
sum_fm = summary(final_model)
model_fit_stats = data.frame(Model = "Final model",
                             Adjusted_R_sq = sum_fm$adj.r.squared,
                             AIC = AIC(final_model), BIC = BIC(final_model))

model_fit_stats %>% gt() %>%
  tab_options(table.font.size = 35) %>% fmt_number(decimals = 3)
```

Model	Adjusted_R_sq	AIC	BIC
Final model	0.743	421.804	458.230

- Another (maybe faster?) way to do it (`glance()` in `broom` package)

```
glance(final_model) %>% mutate(Model = "Final model") %>%
  select(Model, adj.r.squared, AIC, BIC) %>% gt() %>%
  tab_options(table.font.size = 35) %>% fmt_number(decimals = 3)
```

Model	adj.r.squared	AIC	BIC
Final model	0.743	421.804	458.230

Step 6: Assess model fit: Comparing model fits

- Remember the preliminary main effects model (at end of Step 3): same as final model but the continuous varaibles, income and CO2 emissions, were not categorized
- We can compare model fit statistics of the preliminary main effects model and the final model

```r
fm_glance = glance(final_model) %>% mutate(Model = "Final model") %>%
  select(Model, `Adj R-squared` = adj.r.squared, AIC, BIC)
pmem_glance = glance(prelim_me_model) %>%
  mutate(Model = "Preliminary main effects model") %>%
  select(Model, `Adj R-squared` = adj.r.squared, AIC, BIC)
rbind(fm_glance, pmem_glance) %>% gt() %>%
  tab_options(table.font.size = 35) %>% fmt_number(decimals = 3)
```

Model	Adj R-squared	AIC	BIC
Final model	0.743	421.804	458.230
Preliminary main effects model	0.747	417.708	445.028

- Remember, adjusted R^2, AIC, and BIC penalize models for more coefficients
- Preliminary main effects model: better fit statistics, but violates linearity assumption