Lesson 3: Measurement of Association for Contingency Tables

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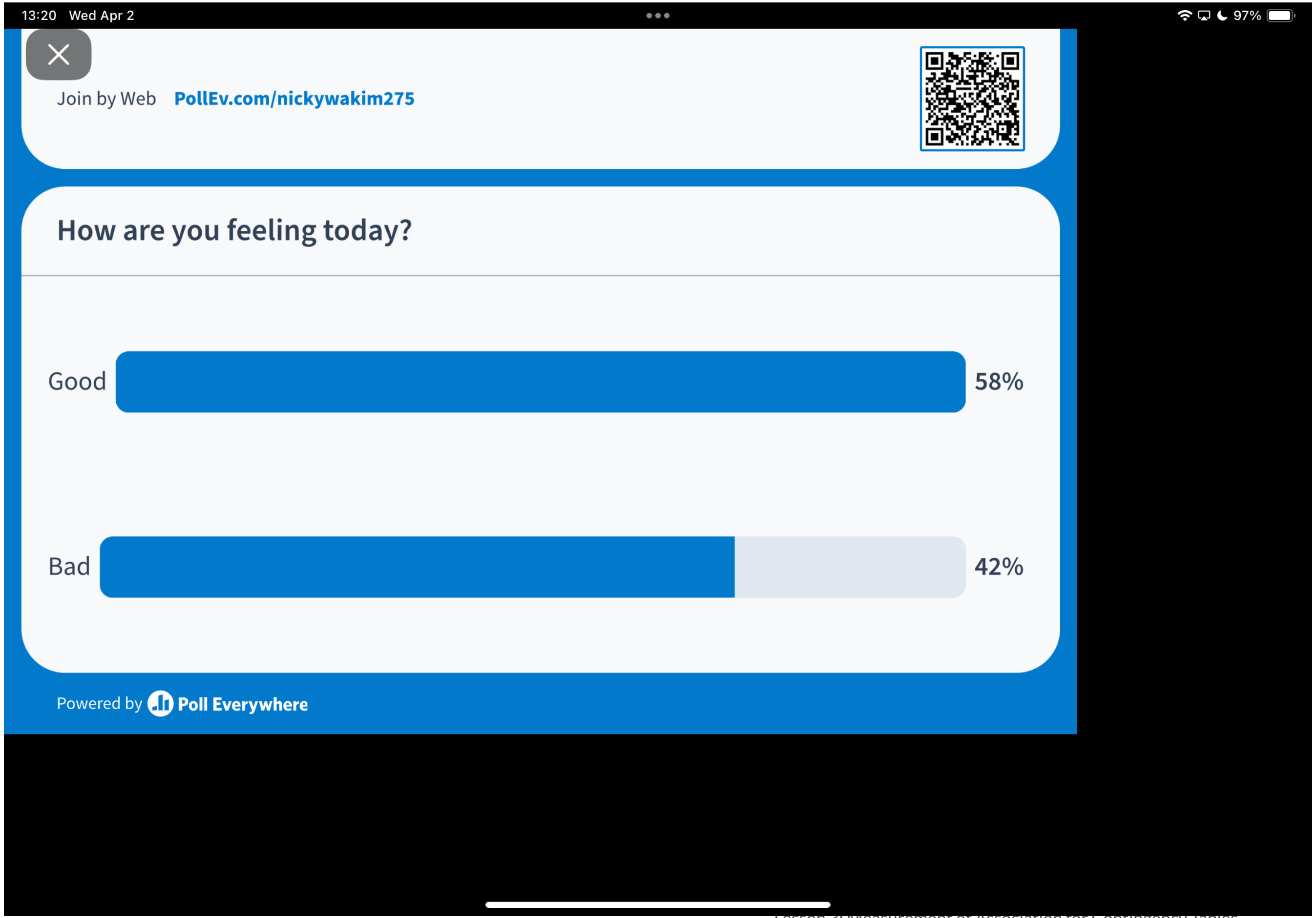
Poll Everywhere Question 1

Make sure to remember your answer!! We'll use this on Wednesday!



Learning Objectives

1. Understand the difference between testing for association and measuring association
2. Estimate the risk difference (and its confidence interval) from a contingency table and interpret the estimate.
3. Estimate the risk ratio (and its confidence interval) from a contingency table and interpret the estimate.
4. Estimate the odds ratio (and its confidence interval) from a contingency table and interpret the estimate.

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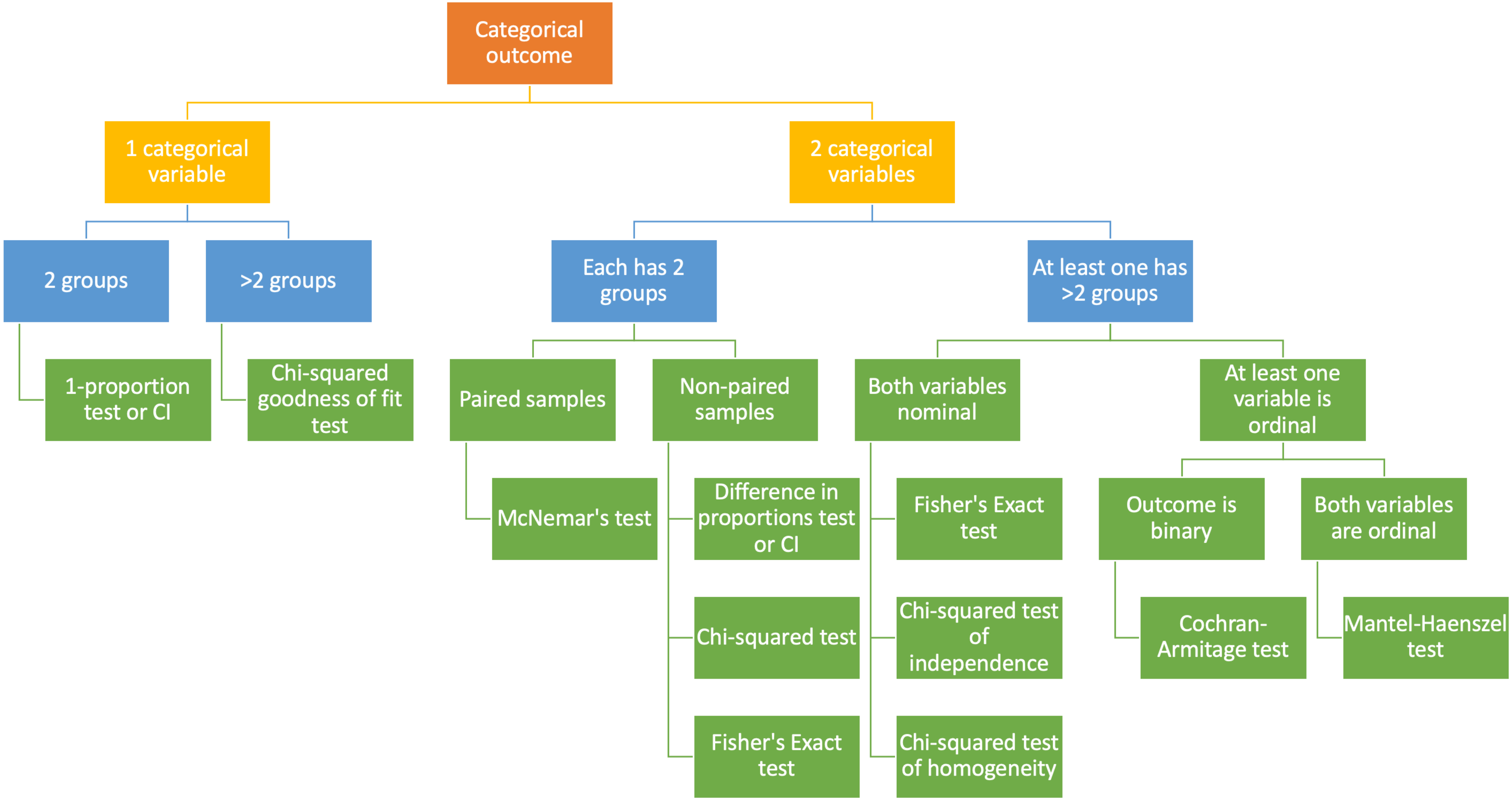
4. Estimate the odds ratio (and its confidence interval) from a contingency table and interpret the estimate.

Review of Test of Association (1/2)

- Last class: learned some tests of association for contingency tables

- For studies with two independent samples
  - General association
    - Chi-squared test
    - Fisher's Exact test
  - Test of trends
    - Cochran-Armitage test
    - Mantel-Haenszel test

Review of Test of Association (2/2)



Test of association *does not* measure association

- Test of association **does not** provide an effective measure of association.

- The p-value alone is not enough
  - $\text{p-value} < 0.05$ suggests there is a statistically significant association between the group and outcome
  - $\text{p-value} = 0.00001$ vs. $\text{p-value} = 0.01$ does not mean the **magnitude** of association is different

- But it does not tell **how different** the risks are between the two groups

- We want a measurement to quantify different risks across the groups

Measures of Association

- When we have a **2x2 contingency table** (binary outcome and binary explanatory variable) and **independent samples**, we have three main options to measure of association:

1. Risk difference (RD)

2. Relative risk (RR)

3. Odds ratio (OR)

Each measures association by comparing the proportion of successes/failures from each categorical group of our explanatory variable.

Before we discuss each further...

Let's define the cells within a 2x2 contingency table:

Explanatory Variable	Response Variable		Total
	Success	Failure	
1	n_{11}	n_{12}	n_1
2	n_{21}	n_{22}	n_2
Total	n_+ (or n_S)	n_- (or n_F)	n

- Then we can define risk: the proportion of "successes"
  - Risk of successful response for explanatory group 1: $\text{Risk}_1 = \dfrac{n_{11}}{n_1}$

↳ Risk or proportion

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Risk Difference (RD)

- Risk difference computes the absolute difference in risk for the two groups (from the explanatory variable)

proportion

- Point estimate:

$$\widehat{RD} = \widehat{p}_1 - \widehat{p}_1 = \frac{n_{11}}{n_1} - \frac{n_{21}}{n_2}$$

  - With range of point estimate from $[-1, 1]$

Explanatory Variable	Response Variable		Total
	Success	Failure	
1	n_{11}	n_{12}	n_1
2	n_{21}	n_{22}	n_2
Total	n_+ (or n_S)	n_- (or n_F)	n

- Approximate standard error:

$$\rightarrow \quad SE_{\widehat{RD}} = \sqrt{\frac{\hat{p}_1 \cdot (1 - \hat{p}_1)}{n_1} + \frac{\hat{p}_2 \cdot (1 - \hat{p}_2)}{n_2}}$$

- 95% Wald confidence interval for $\widehat{RD}$: → given expected cell counts > 5

$$\widehat{RD} \pm 1.96 \cdot SE_{\widehat{RD}}$$

Recall the Strong Heart Study

The Strong Heart Study is an ongoing study of American Indians residing in 13 tribal communities in three geographic areas (AZ, OK, and SD/ND). We will look at data from this study examining the **incidence of diabetes** at a follow-up visit and **impaired glucose tolerance** (ITG) at baseline (4 years apart).

Glucose tolerance	Diabetes		
	No	Yes	Total
Impaired	334	198	532
Normal	1004	128	1132
Total	1338	326	1664

SHS Example: Risk Difference

Glucose tolerance	Diabetes: No	Diabetes: Yes	Total
Impaired	334	198	532
Normal	1004	128	1132
Total	1338	326	1664

Risk difference

Compute the point estimate and 95% confidence interval for the diabetes risk difference between impaired and normal glucose tolerance.

Needed steps:

0. expected cells count > 5
1. Compute the risk difference
2. Compute 95% confidence interval
3. Interpret the estimate

SHS Example: Risk Difference (1/4)

Risk difference

Compute the point estimate and 95% confidence interval for the diabetes risk difference between impaired and normal glucose tolerance.

Glucose tolerance	Diabetes		Total
	No	Yes	
Impaired	334	198	532
Normal	1004	128	1132
Total	1338	326	1664

1. Compute the risk difference

$$\widehat{RD} = \hat{p}_1 - \hat{p}_2 = \frac{n_{11}}{n_1} - \frac{n_{21}}{n_2} = \frac{198}{532} - \frac{128}{1132} = 0.3722 - 0.1131 = 0.2591$$

SHS Example: Risk Difference (2/4)

Risk difference

Compute the point estimate and 95% confidence interval for the diabetes risk difference between impaired and normal glucose tolerance.

Glucose tolerance	Diabetes		
	No	Yes	Total
Impaired	334	198	532
Normal	1004	128	1132
Total	1338	326	1664

2. Compute 95% confidence interval

$$
\begin{aligned}
&\widehat{RD} \pm z^*_{(1-\frac{\alpha}{2})} \times SE_{\widehat{RD}} \\
=&\widehat{RD} \pm z^*_{(1-\frac{\alpha}{2})} \times \sqrt{\frac{\hat{p}_1\left(1-\hat{p}_1\right)}{n_1}+\frac{\hat{p}_2(1-\hat{p}_2)}{n_2}} \\
=&0.2591 \pm 1.96 \times \sqrt{\frac{0.3722(1-0.3722)}{532}+\frac{0.1131(1-0.1131)}{1132}} \\
=&(0.2141,\ 0.3041)
\end{aligned}
$$

SHS Example: Risk Difference (3/4)

Risk difference

Compute the point estimate and 95% confidence interval for the diabetes risk difference between impaired and normal glucose tolerance.

Glucose tolerance	Diabetes		
	No	Yes	Total
Impaired	334	198	532
Normal	1004	128	1132
Total	1338	326	1664

1/2. Compute risk difference and 95% confidence interval

```r
fmsb::riskdifference(198, 128, 532, 1132)
```

```
                Cases People at risk      Risk
Exposed   198.0000000    532.0000000 0.3721805
Unexposed 128.0000000   1132.0000000 0.1130742
Total     326.0000000   1664.0000000 0.1959135

    Risk difference and its significance probability (H0: The difference
    equals to zero)

data:  198 128 532 1132
p-value < 2.2e-16
95 percent confidence interval:
 0.2140779 0.3041346
sample estimates:
[1] 0.2591062
```

→ 95% CI

RD

SHS Example: Risk Difference (4/4)

Risk difference

Compute the point estimate and 95% confidence interval for the diabetes risk difference between impaired and normal glucose tolerance.

Glucose tolerance	Diabetes No	Diabetes Yes	Total
Impaired	334	198	532
Normal	1004	128	1132
Total	1338	326	1664

3. Interpret the estimate

The diabetes diagnosis risk difference between impaired and normal glucose tolerance is 0.2591 (95% CI: 0.2141, 0.3041). Since the 95% confidence interval does not contain 0, we have sufficient evidence that the risk of diabetes diagnosis ~~within 4 year follow-up~~ for people with impaired versus normal glucose tolerance is different.

When is the risk difference misleading?

- The same risk differences can have very different clinical meanings depending on the risk for each group

$$\hat{RD} = 0.009 = \hat{p}_1 - \hat{p}_2$$

- **Example:** for two treatments A and B, we know the risk difference (RD) is 0.009. Is it a meaningful difference?
  - If the risk is 0.01 for Trt A and 0.001 for Trt B?
  - If the risk is 0.41 for Trt A and 0.401 for Trt B?

- Using the RD alone to summarize the difference in risks for comparing the two groups can be **misleading**
  - The **ratio of risk** can provide an informative descriptive measure of the "relative risk"

Learning Objectives

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4. Estimate the odds ratio (and its confidence interval) from a contingency table and interpret the estimate.

Relative Risk (RR)

- Relative risk computes the **ratio** of each group's proportions of "success"
  - Also called **risk ratio**

- Point estimate:

$$\widehat{RR} = \frac{\hat{p}_1}{\hat{p}_2} = \frac{n_{11}/n_1}{n_{21}/n_2}$$

  - Range: $[0, \infty]$

Explanatory Variable	Response Variable		Total
	Success	Failure	
1	n_{11}	n_{12}	n_1
2	n_{21}	n_{22}	n_2
Total	n_+ (or n_S)	n_- (or n_F)	n

if $\hat{p}_1 = 0$
regardless of $\hat{p}_2$ $\widehat{RR} = 0$

if $\hat{p}_2 = 0$
regardless of $\hat{p}_1 \to \widehat{RR} = \infty$

$\hat{p}_1 = \hat{p}_2 \Rightarrow \widehat{RR} = 1$

$\hat{p}_1 > \hat{p}_2 \Rightarrow \widehat{RR} > 1$

$\hat{p}_2 > \hat{p}_1 \Rightarrow \widehat{RR} < 1$

Poll Everywhere Question 2

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If we use the previous example with two scenarios with the same risk difference. What is the relative risk for each scenario:
1. The risk is 0.01 for Trt A and 0.001 for Trt B
2. The risk is 0.41 for Trt A and 0.401 for Trt B.

4 0

10 and 1.02

2 0

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scenario 1:

$$\widehat{RR} = \frac{0.01}{0.001} = 10$$

Scenario 2:

$$\widehat{RR} = \frac{0.41}{0.401} = 1.02$$

Log-transformation of RR

$\hat{p}_2 > \hat{p}_1$ $[0, 1)$ $\hat{p}_1 > \hat{p}_2$ $(1, \infty)$

- Sampling distribution of the relative risk is **highly skewed** unless sample sizes are quite large
  - **Log transformation** results in *approximately normal distribution*
  - Thus, **compute confidence interval using normally distributed, log-transformed RR**
  - Then we convert back to the RR inverse log.
- We take the log (natural log) of RR: $\ln(\widehat{RR})$ or $log(\widehat{RR})$
  - Whenever I say "log" I mean natural log (base e, very common in statistics)
- Then we need to find approximate standard error for $\ln(\widehat{RR})$

$$SE_{\ln(\widehat{RR})} = \sqrt{\frac{1}{n_{11}} - \frac{1}{n_1} + \frac{1}{n_{21}} - \frac{1}{n_2}}$$

Explanatory Variable	Response Variable		Total
	Success	Failure	
1	n_{11}	n_{12}	n_1
2	n_{21}	n_{22}	n_2
Total	n_+ (or n_S)	n_- (or n_F)	n

- 95% confidence interval for $\ln(\widehat{RR})$:

$$\ln(\widehat{RR}) \pm 1.96 \times SE_{\ln(\widehat{RR})}$$

How do we get back to the RR scale?

- We **computed confidence interval** using normally distributed, log-transformed RR ($\ln(\widehat{RR})$):

LCI UCI

$$\left(\ln(\widehat{RR}) - 1.96 \times SE_{\ln(\widehat{RR})}, \ln(\widehat{RR}) + 1.96 \times SE_{\ln(\widehat{RR})}\right)$$

inverse log.

- Now we need to exponentiate the CI to get back to interpretable values
  - **Take exponential of lower and upper bounds**
- 95% confidence interval for RR: two ways to display equation

$$\left(e^{\ln(\widehat{RR}) - 1.96 \times SE_{\ln(\widehat{RR})}}, e^{\ln(\widehat{RR}) + 1.96 \times SE_{\ln(\widehat{RR})}}\right)$$

$$\left(\exp\left[\ln(\widehat{RR}) - 1.96 \times SE_{\ln(\widehat{RR})}\right], \exp\left[\ln(\widehat{RR}) + 1.96 \times SE_{\ln(\widehat{RR})}\right]\right)$$

Relative Risk (RR)

- Can you compute the estimated RRs for the previous example?
  - If the risk for Trt A is 0.01 and Trt B is 0.001? $\widehat{RR} = 10$
  - If the risk for Trt A is 0.41 and Trt B is 0.401? $\widehat{RR} = 1.02$

- When $\widehat{RR} = 1$ ...
  - Risk is the same for the two groups
  - In other words, the **group and the outcome are independent**

- When computing $\widehat{RR}$ it is important to **identify which variable is the response variable and which is explanatory variable**
  - We may say "risk for Trt A" but this translates to the risk (or probability) of outcome success for those receiving Trt A

SHS Example: Relative Risk (1/6)

Relative risk

Compute the point estimate and 95% confidence interval for the diabetes Relative risk between impaired and normal glucose tolerance.

Glucose tolerance	Diabetes: No	Diabetes: Yes	Total
Impaired	334	198	532
Normal	1004	128	1132
Total	1338	326	1664

Needed steps:

1. Compute the relative risk
2. Find confidence interval of log RR
3. Convert back to RR
4. Interpret the estimate

SHS Example: Relative Risk (2/6)

Relative risk

Compute the point estimate and 95% confidence interval for the diabetes Relative risk between impaired and normal glucose tolerance.

Glucose tolerance	Diabetes		
	No	Yes	Total
Impaired	334	198	532
Normal	1004	128	1132
Total	1338	326	1664

1. Compute the relative risk

$$\widehat{RR} = \frac{\hat{p}_1}{\hat{p}_2} = \frac{n_{11}/n_1}{n_{21}/n_2} = \frac{198/532}{128/1132} = \frac{0.3722}{0.1131} = 3.2915$$

comparing impaired vs. normal glucose

SHS Example: Relative Risk (3/6)

Relative risk

Compute the point estimate and 95% confidence interval for the diabetes Relative risk between impaired and normal glucose tolerance.

Glucose tolerance	Diabetes		
	No	Yes	Total
Impaired	334	198	532
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Total	1338	326	1664

95%

2. Find confidence interval of log RR

ln (RR) ~ Normal

$$\ln(\widehat{RR}) \pm 1.96 \times SE_{\ln(\widehat{RR})}$$

$$= \ln(\widehat{RR}) \pm z^*_{(1-\frac{\alpha}{2})} \times \sqrt{\frac{1}{n_{11}} - \frac{1}{n_1} + \frac{1}{n_{21}} - \frac{1}{n_2}}$$

$$= 1.1913 \pm 1.96 \times \sqrt{\frac{1}{198} - \frac{1}{532} + \frac{1}{128} - \frac{1}{1132}}$$

$$= (0.9944,\ 1.3883)$$

on log scale

SHS Example: Relative Risk (4/6)

Relative risk

Compute the point estimate and 95% confidence interval for the diabetes Relative risk between impaired and normal glucose tolerance.

Glucose tolerance	Diabetes: No	Diabetes: Yes	Total
Impaired	334	198	532
Normal	1004	128	1132
Total	1338	326	1664

3. Convert back to RR

$$
\begin{aligned}
&(\exp(0.9944),\ \exp(1.3883)) \\
=&(2.703,\ 4.0081)
\end{aligned}
$$

SHS Example: Relative Risk (5/6)

Relative risk

Compute the point estimate and 95% confidence interval for the diabetes Relative risk between impaired and normal glucose tolerance.

Glucose tolerance	Diabetes: No	Diabetes: Yes	Total
Impaired	334	198	532
Normal	1004	128	1132
Total	1338	326	1664

1/2/3. Compute risk ratio and 95% confidence interval

```
library(epitools)
SHS_ct = table(SHS$glucimp, SHS$case)
riskratio(x = SHS_ct, rev = "rows")$measure
```

```
          risk ratio with 95% C.I.
           estimate    lower    upper
Normal     1.000000       NA       NA
Impaired   3.291471 2.702998 4.008061
```

SHS:

individual	glucimp	case
1	0	1
2	1	1
3	0	0
4	0	0

Pause: other option in pubh package

```
SHS = SHS %>% mutate(glucimp = as.factor(glucimp) %>% relevel(ref = "Normal"))
contingency(case ~ glucimp, data = SHS)
```

each individual row

```
           Outcome
Predictor     1    0
  Impaired  198  334
  Normal    128 1004

             Outcome +    Outcome -      Total                 Inc risk *
Exposed +          198          334        532     37.22 (33.10 to 41.48)
Exposed -          128         1004       1132      11.31 (9.52 to 13.30)
Total              326         1338       1664     19.59 (17.71 to 21.58)

Point estimates and 95% CIs:
-------------------------------------------------------------------
Inc risk ratio                                 3.29 (2.70, 4.01)
Inc odds ratio                                 4.65 (3.61, 6.00)
Attrib risk in the exposed *                   25.91 (21.41, 30.41)
Attrib fraction in the exposed (%)            69.62 (63.00, 75.05)
Attrib risk in the population *                8.28 (5.63, 10.94)
Attrib fraction in the population (%)         42.28 (34.71, 48.98)
-------------------------------------------------------------------
Uncorrected chi2 test that OR = 1: chi2(1) = 154.239 Pr>chi2 = <0.001
Fisher exact test that OR = 1: Pr>chi2 = <0.001
 Wald confidence limits
 CI: confidence interval
 * Outcomes per 100 population units


    Pearson's Chi-squared test with Yates' continuity correction

data:  dat
X-squared = 152.6, df = 1, p-value < 2.2e-16
```

SHS Example: Relative Risk (6/6)

Relative risk

Compute the point estimate and 95% confidence interval for the diabetes Relative risk between impaired and normal glucose tolerance.

Glucose tolerance	Diabetes No	Diabetes Yes	Total
Impaired	334	198	532
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Total	1338	326	1664

3. Interpret the estimate

$\widehat{RR}$

risk of outcome

grp 1

The estimated risk of diabetes is 3.29 times greater for American Indians who had impaired glucose tolerance at baseline compared to those who had normal glucose tolerance (95% CI: 2.70, 4.01).

grp 2

Additional interpretation of 95% CI (not needed): We are 95% confident that the (population) relative risk is between 2.70 and 4.01.

Since the 95% confidence interval does not include 1, there is sufficient evidence that the risk of diabetes differs significantly between impaired and normal glucose tolerance at baseline.

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Odds (building up to Odds Ratio)

- For a probability of success p (or sometimes referred to as π), the odds of success is:

for exp var grp 1:

$$\widehat{\text{odds}} = \frac{\hat{p}}{1-\hat{p}} = \frac{\hat{\pi}}{1-\hat{\pi}}$$

  - **Example:** if $\hat{\pi} = 0.75$, then odds of success $= \frac{0.75}{0.25} = 3$

- If odds > 1, it implies a success is more likely than a failure
  - Example: for $\widehat{\text{odds}} = 3$, we expect to observe three times as many successes as failures
- If odds is known, the probability of success can be computed

$$\hat{\pi} = \frac{\widehat{\text{odds}}}{\widehat{\text{odds}}+1}$$

Explanatory Variable	Response Variable		Total
	Success	Failure	
1	n_{11}	n_{12}	n_1
2	n_{21}	n_{22}	n_2
Total	n_+ (or n_S)	n_- (or n_F)	n

$$\hat{p}_1 = \frac{n_{11}}{n_1}$$

$$1-\hat{p}_1 = 1 - \frac{n_{11}}{n_1} = \frac{n_1}{n_1} - \frac{n_{11}}{n_1}$$

$$\frac{n_1}{n_1} = 1$$

$$\widehat{odds} = \frac{\hat{p}}{1-\hat{p}}$$

$$\widehat{odds}(1-\hat{p}) = \hat{p}$$

$$\widehat{odds} - \hat{p}\,\widehat{odds} = \hat{p}$$

$$+\hat{p}\,\widehat{odds} \qquad +\hat{p}\,\widehat{odds}$$

$$\widehat{odds} = \hat{p} + \hat{p}\,\widehat{odds}$$

$$\frac{\widehat{odds}}{1+\widehat{odds}} = \frac{\hat{p}(1+\widehat{odds})}{1+\widehat{odds}}$$

Odds Ratio (OR)

- Odds ratio is the ratio of two odds:

$$\widehat{OR} = \frac{\widehat{\text{odds}}_1}{\widehat{\text{odds}}_2} = \frac{\hat{p}_1/(1-\hat{p}_1)}{\hat{p}_2/(1-\hat{p}_2)}$$

Explanatory Variable	Response Variable		Total
	Success	Failure	
1	n_{11}	n_{12}	n_1
2	n_{21}	n_{22}	n_2
Total	n_+ (or n_S)	n_- (or n_F)	n

  - **Range:** $[0, \infty]$

$\widehat{odds}_1 > \widehat{odds}_2 \Rightarrow \widehat{OR} > 1$

  - **Interpretation:** The odds of success for "group 1" is "$\widehat{OR}$" times the odds of success for "group 2"

- What do values of odds ratios mean?

Odds Ratio	Clinical Meaning
$\widehat{OR} < 1$	Odds of success is smaller in group 1 than in group 2
$\widehat{OR} = 1$	Explanatory and response variables are independent
$\widehat{OR} > 1$	Odds of success is greater in group 1 than in group 2

$\widehat{odds}_1 < \widehat{odds}_2$

$\widehat{odds}_1 = \widehat{odds}_2$

$\widehat{odds}_1 > \widehat{odds}_2$

Poll Everywhere Question 3

14:18 Wed Apr 2 ••• 80%

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Given $p_2 = 0.8$, what is the range of possible values for our relative risk and our odds ratio?

RR: ~~[0, ∞)~~, OR: [0, ∞)	19%
RR: [0, 1.25], OR: [0, ∞) ✓	24%
RR: ~~[0, ∞)~~, OR: [0, 1.25]	38%
$RR : [0, 1.25], OR : [0, 1.25]$	19%

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$$\hat{RR} = \frac{\hat{p}_1}{\hat{p}_2} = \frac{\hat{p}_1}{0.8}$$

$$\hat{p}_1 \in [0, 1]$$

$$\hat{p}_1 = 0 \Rightarrow \hat{RR} = 0$$

$$\hat{p}_1 = 1 \Rightarrow \hat{RR} = 1.25$$

$$\hat{OR} = \frac{\frac{\hat{p}_1}{1-\hat{p}_1}}{\frac{\hat{p}_2}{1-\hat{p}_2}} \quad \frac{\hat{p}_2}{1-\hat{p}_2} = \frac{0.8}{0.2}$$

$$= \frac{\frac{\hat{p}_1}{1-\hat{p}_1}}{4}$$

$$\hat{p}_1 = 0 \Rightarrow \frac{0}{1-0} = \frac{0}{1}$$

$$\hat{p}_1 = 1 \Rightarrow \frac{1}{1-1} = \infty$$

Odds Ratio (OR)

- Values of OR farther from 1.0 in a given direction represent stronger association
  - An OR = 4 is farther from independence than an OR = 2
  - An OR = 0.25 is farther from independence than an OR = 0.5
  - For OR = 4 and OR = 0.25, they are equally away from independence (because ¼ = 0.25)

4 is stronger assoc. than 2

0.25 is stronger assoc. than 0.5

- We take the inverse of the OR for **success** of group 1 compared to group 2 to get…
  - OR for **failure** of group 1 compared to group 2
  - OR for **success** of group 2 compared to group 1

$$OR_a = \frac{\frac{\hat{p}_1}{1-\hat{p}_1}}{\frac{\hat{p}_2}{1-\hat{p}_2}}$$

$$\frac{1}{OR_a} = \frac{1}{\left[\frac{\frac{\hat{p}_1}{1-\hat{p}_1}}{\frac{\hat{p}_2}{1-\hat{p}_2}}\right]} = \frac{\frac{1-\hat{p}_1}{\hat{p}_1}}{\frac{1-\hat{p}_2}{\hat{p}_2}}$$

$$\frac{1}{OR_a} = \frac{1}{\left[\frac{\frac{\hat{p}_1}{1-\hat{p}_1}}{\frac{\hat{p}_2}{1-\hat{p}_2}}\right]} = \frac{\frac{\hat{p}_2}{1-\hat{p}_2}}{\frac{\hat{p}_1}{1-\hat{p}_1}}$$

Log-transformation of *OR*

- Like RR, sampling distribution of the odds ratio is highly skewed
  - Log transformation results in approximately normal distribution
  - Thus, compute confidence interval using normally distributed, log-transformed OR

- Approximate standard error for $\ln(\widehat{OR})$:

$$SE_{\ln(\widehat{OR})} = \sqrt{\frac{1}{n_{11}} + \frac{1}{n_{12}} + \frac{1}{n_{21}} + \frac{1}{n_{22}}}$$

- 95% confidence interval for $\ln(\widehat{OR})$:

$$\ln(\widehat{OR}) \pm 1.96 \times SE_{\ln(\widehat{OR})}$$

z-score @ 95%

Explanatory Variable	Response Variable		Total
	Success	Failure	
1	n_{11}	n_{12}	n_1
2	n_{21}	n_{22}	n_2
Total	n_+ (or n_S)	n_- (or n_F)	n

How do we get back to the OR scale?

- We computed confidence interval using normally distributed, log-transformed OR ($\ln(\widehat{OR})$):

$$\left(\ln(\widehat{OR}) - 1.96 \times SE_{\ln(\widehat{OR})}, \ \ln(\widehat{OR}) + 1.96 \times SE_{\ln(\widehat{OR})}\right)$$

- Now we need to **exponentiate the CI to get back to interpretable values**
  - Take exponential of lower and upper bounds
- 95% confidence interval for RR: two ways to display equation

$$\left(e^{\ln(\widehat{OR}) - 1.96 \times SE_{\ln(\widehat{OR})}}, \ e^{\ln(\widehat{OR}) + 1.96 \times SE_{\ln(\widehat{OR})}}\right)$$

$$\left(\exp\left[\ln(\widehat{OR}) - 1.96 \times SE_{\ln(\widehat{OR})}\right], \ \exp\left[\ln(\widehat{OR}) + 1.96 \times SE_{\ln(\widehat{OR})}\right]\right)$$

SHS Example: Odds Ratio (1/6)

Odds ratio

Compute the point estimate and 95% confidence interval for the diabetes odds ratio between impaired and normal glucose tolerance.

Glucose tolerance	Diabetes		
	No	Yes	Total
Impaired	334	198	532
Normal	1004	128	1132
Total	1338	326	1664

Needed steps:

0. expected cell counts > 5

1. Compute the odds ratio
2. Find confidence interval of log OR
3. Convert back to OR
4. Interpret the estimate

SHS Example: Odds Ratio (2/6)

Odds ratio

Compute the point estimate and 95% confidence interval for the diabetes Odds ratio between impaired and normal glucose tolerance.

Glucose tolerance	Diabetes		
	No	Yes	Total
Impaired	334	198	532
Normal	1004	128	1132
Total	1338	326	1664

1. Compute the odds ratio

$$\widehat{p}_1 = 198/532 = 0.3722, \widehat{p}_2 = 128/1132 = 0.1131$$

$$\widehat{OR} = \frac{\widehat{p}_1/(1-\widehat{p}_1)}{\widehat{p}_2/(1-\widehat{p}_2)} = \frac{0.3722/(1-0.3722)}{0.1131/(1-0.1131)} = 4.6499$$

SHS Example: Odds Ratio (3/6)

Odds ratio

Compute the point estimate and 95% confidence interval for the diabetes Odds ratio between impaired and normal glucose tolerance.

Glucose tolerance	Diabetes No	Diabetes Yes	Total
Impaired	334	198	532
Normal	1004	128	1132
Total	1338	326	1664

2. Find confidence interval of log OR

$$
\begin{aligned}
&\ln(\widehat{OR}) \pm 1.96 \times SE_{\ln(\widehat{OR})} \\
=&\ln(\widehat{OR}) \pm z^*_{(1-\frac{\alpha}{2})} \times \sqrt{\frac{1}{n_{11}} + \frac{1}{n_{12}} + \frac{1}{n_{21}} + \frac{1}{n_{22}}} \\
=&1.5368 \pm 1.96 \times \sqrt{\frac{1}{198} + \frac{1}{334} + \frac{1}{128} + \frac{1}{1004}} \\
=&(1.2824,\ 1.7913)
\end{aligned}
$$

SHS Example: Odds Ratio (4/6)

Odds ratio

Compute the point estimate and 95% confidence interval for the diabetes Odds ratio between impaired and normal glucose tolerance.

Glucose tolerance	Diabetes: No	Diabetes: Yes	Total
Impaired	334	198	532
Normal	1004	128	1132
Total	1338	326	1664

3. Convert back to OR

$$
\begin{aligned}
&(\exp(1.2824),\ \exp(1.7913)) \\
=&(3.6053,\ 5.9971)
\end{aligned}
$$

$\hat{OR} = 4.65$

SHS Example: Odds Ratio (5/6)

Odds ratio

Compute the point estimate and 95% confidence interval for the diabetes Odds ratio between impaired and normal glucose tolerance.

Glucose tolerance	Diabetes		
	No	Yes	Total
Impaired	334	198	532
Normal	1004	128	1132
Total	1338	326	1664

1/2/3. Compute OR and 95% confidence interval

```
library(epitools)
SHS_ct = table(SHS$glucimp, SHS$case)
# no `rev` needed below bc we set the reference level in slide 32
oddsratio(x = SHS_ct, method = "wald")$measure
```

```
         odds ratio with 95% C.I.
          estimate    lower    upper
Normal    1.000000       NA       NA
Impaired  4.649888 3.605289 5.997148
```

Normal approx

rev= "rows"

Pause: other option in pubh package

```r
contingency(case ~ glucimp, data = SHS, digits = 3)
```

```
          Outcome
Predictor     1    0
  Impaired  198  334
  Normal    128 1004

             Outcome +    Outcome -      Total                 Inc risk *
Exposed +          198          334        532   37.218 (33.097 to 41.482)
Exposed -          128         1004       1132    11.307 (9.521 to 13.298)
Total              326         1338       1664   19.591 (17.709 to 21.581)

Point estimates and 95% CIs:
-------------------------------------------------------------------
Inc risk ratio                                 3.291 (2.703, 4.008)
Inc odds ratio                                 4.650 (3.605, 5.997)
Attrib risk in the exposed *                   25.911 (21.408, 30.413)
Attrib fraction in the exposed (%)            69.618 (63.004, 75.050)
Attrib risk in the population *                8.284 (5.631, 10.937)
Attrib fraction in the population (%)         42.284 (34.713, 48.976)
-------------------------------------------------------------------
Uncorrected chi2 test that OR = 1: chi2(1) = 154.239 Pr>chi2 = <0.001
Fisher exact test that OR = 1: Pr>chi2 = <0.001
 Wald confidence limits
 CI: confidence interval
 * Outcomes per 100 population units


    Pearson's Chi-squared test with Yates' continuity correction

data:  dat
X-squared = 152.6, df = 1, p-value < 2.2e-16
```

SHS Example: Odds Ratio (6/6)

Odds ratio

Compute the point estimate and 95% confidence interval for the diabetes Odds ratio between impaired and normal glucose tolerance.

Glucose tolerance	Diabetes		
	No	Yes	Total
Impaired	334	198	532
Normal	1004	128	1132
Total	1338	326	1664

3. Interpret the estimate

The estimated odds of diabetes for American Indians with impaired glucose tolerance at baseline is 4.65 times the odds for American Indians with normal glucose tolerance at baseline. (95% CI: 3.61, 6.00)

Additional interpretation of 95% CI (not needed): We are 95% confident that the odds ratio is between 3.61 and 6.00.

Since the 95% confidence interval does not include 1, there is sufficient evidence that the odds of diabetes differs significantly between impaired and normal glucose tolerance at baseline.

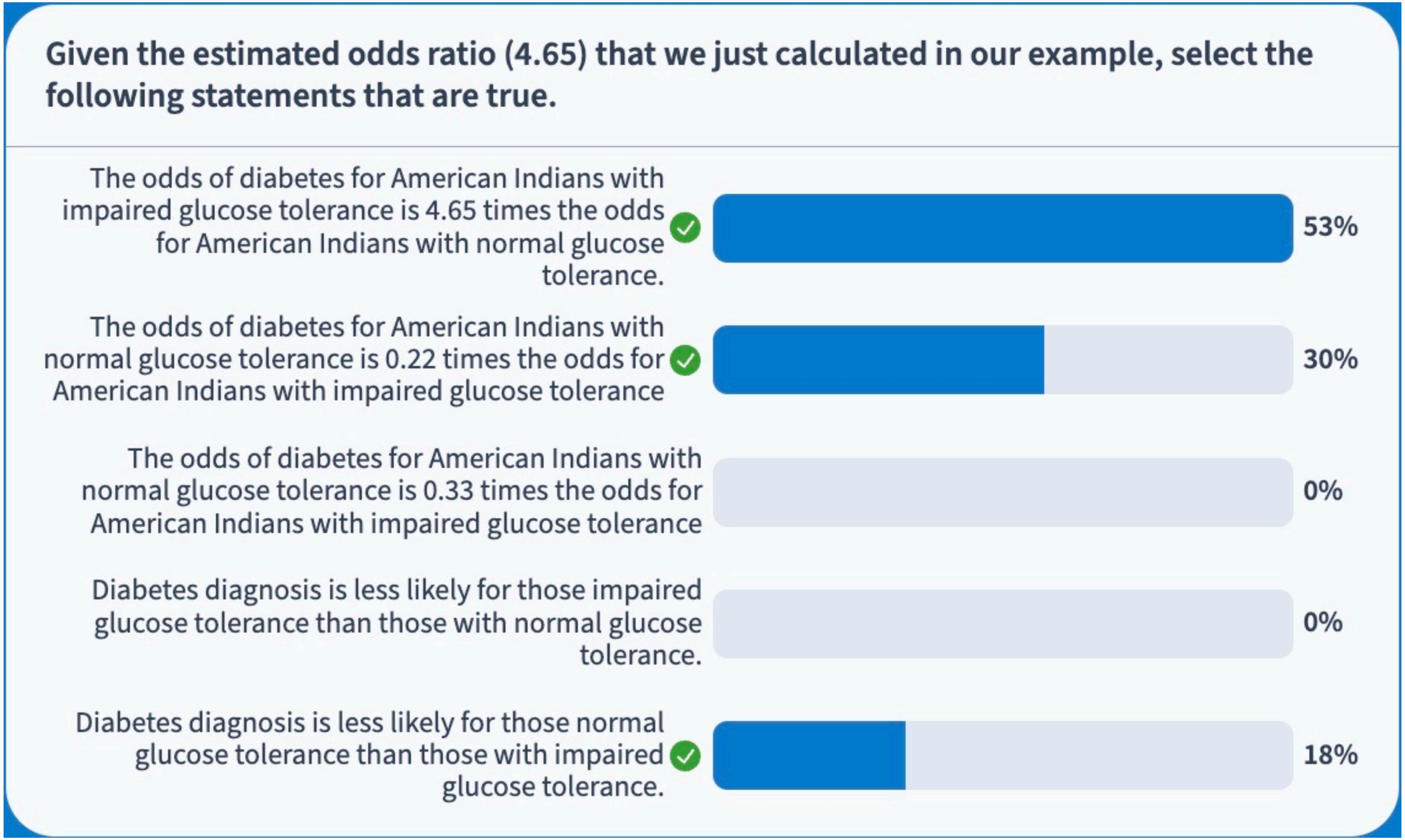
Inversing an Odds Ratio

- Some people prefer interpretations of OR > 1 instead of an OR < 1
- The transformation can easily be done by inverse
  - Remember we discussed that $OR = 4$ is an equivalent a strong association as $OR = 0.25$ (1/4)

- OR comparing group 1 to group 2 = inverse of OR comparing group 2 to group 1

$$OR_{1v2} = \frac{\hat{p}_1/(1-\hat{p}_1)}{\hat{p}_2/(1-\hat{p}_2)} = \frac{1}{\frac{\hat{p}_2/(1-\hat{p}_2)}{\hat{p}_1/(1-\hat{p}_1)}} = \frac{1}{OR_{2v1}}$$

Poll Everywhere Question 4



$\frac{1}{4.65}$

SHS Example: Inversing Odds Ratio

Inversing odds ratio

Compute the point estimate and 95% confidence interval for the diabetes odds ratio between **normal** and **impaired** glucose tolerance.

Glucose tolerance	Diabetes No	Diabetes Yes	Total
Impaired	334	198	532
Normal	1004	128	1132
Total	1338	326	1664

Needed steps:

1. Inverse point estimate and confidence interval

$$\widehat{OR} = \frac{1}{4.6499} = 0.2151$$

The 95% Confidence interval is then

$$\left(\frac{1}{5.9971}, \frac{1}{3.6053}\right) = (0.1667, 0.2774)$$

SHS Example: Inversing Odds Ratio

Inversing odds ratio

Compute the point estimate and 95% confidence interval for the diabetes odds ratio between **normal** and **impaired** glucose tolerance.

Glucose tolerance	Diabetes No	Diabetes Yes	Total
Impaired	334	198	532
Normal	1004	128	1132
Total	1338	326	1664

Needed steps:

1. Inverse point estimate and confidence interval

```
library(epitools)
oddsratio(x = SHS_ct, method = "wald", rev = "rows")$measure
```

```
         odds ratio with 95% C.I.
          estimate     lower     upper
Impaired 1.000000        NA        NA
Normal   0.215059 0.1667459 0.2773702
```

SHS Example: Inversing Odds Ratio

Inversing odds ratio

Compute the point estimate and 95% confidence interval for the diabetes odds ratio between **normal** and **impaired** glucose tolerance.

Glucose tolerance	Diabetes: No	Diabetes: Yes	Total
Impaired	334	198	532
Normal	1004	128	1132
Total	1338	326	1664

Needed steps:

2. Interpret the estimate

The estimated odds of diabetes for American Indians with normal glucose tolerance at baseline is 0.22 times the odds for American Indians with impaired glucose tolerance at baseline.

Additional interpretation of 95% CI (not needed): We are 95% confident that the odds ratio is between 0.17 and 0.28.

Since the 95% confidence interval does not include 1, there is sufficient evidence that the odds of diabetes differs significantly between impaired and normal glucose tolerance at baseline.

Learning Objectives

1. Understand the difference between testing for association and measuring association
2. Estimate the risk difference (and its confidence interval) from a contingency table and interpret the estimate.
3. Estimate the risk ratio (and its confidence interval) from a contingency table and interpret the estimate.
4. Estimate the odds ratio (and its confidence interval) from a contingency table and interpret the estimate.

`pubh` vs. `epitools`

- In `pubh` with `contingency()`
  - Get all the info at once
  - Really nice to double check how the code is interpreting your input
- In `epitools` with `riskratio()` or `oddsratio()`
  - Much easier to grab the numbers!
  - In Quarto you can take R code and directly put it in your text

```
g = oddsratio(x = SHS_ct, method = "wald", rev = "rows")
g$measure[2,1]
```

```
[1] 0.215059
```

  - I can write `{r eval="false" echo="true"} round(g$measure[2,1], 3)` to print the number 0.215