# Lesson 6: Interpretations and Visualizations of Odds Ratios

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# Learning Objectives

1. Interpret odds ratios from fitted simple logistic regression model for a continuous explanatory variable.
2. Interpret odds ratios from fitted simple logistic regression model for a binary explanatory variable.
3. Interpret odds ratios from fitted simple logistic regression model for a multi-level categorical explanatory variable.
4. Report the odds ratio using a table and/or a forest plot.

# Recall our example: Late stage breast cancer diagnosis

- Recall that we fitted a simple logistic regression for late stage breast cancer diagnosis using the predictor, age:

```
bc_reg = bc %>% glm(formula = Late_stage_diag ~ Age_c, family = binomial)
tidy(bc_reg, conf.int=T) %>% gt() %>% tab_options(table.font.size = 38) %>%
  fmt_number(decimals = 3)
```

| term | estimate | std.error | statistic | p.value | conf.low | conf.high |
|---|---|---|---|---|---|---|
| (Intercept) | −0.989 | 0.023 | −42.637 | 0.000 | −1.035 | −0.944 |
| Age_c | 0.057 | 0.003 | 17.780 | 0.000 | 0.051 | 0.063 |

**Population logistic regression model**

$$\text{logit}(\pi(Age)) = \beta_0 + \beta_1 \cdot Age$$

**Fitted logistic regression model**

$$\text{logit}(\widehat{\pi}(Age)) = -0.989 + 0.057 \cdot Age$$

# Coefficients on the log-odds scale

## Population logistic regression model

$$\text{logit}(\pi(X)) = \beta_0 + \beta_1 \cdot X$$

- $\beta_0$: log-odds of $Y = 1$ when $X$ is 0
- $\beta_1$: increase in log-odds of $Y = 1$ for every 1 unit increase in $X$

## Fitted logistic regression model

$$\text{logit}(\widehat{\pi}(X)) = \widehat{\beta}_0 + \widehat{\beta}_1 \cdot X$$

- $\widehat{\beta}_0$: estimated log-odds of $Y = 1$ when $X$ is 0
- $\widehat{\beta}_1$: estimated increase in log-odds of $Y = 1$ for every 1 unit increase in $X$
- Can use expected instead of estimated

# Recall our example: Late stage breast cancer diagnosis

- Fitted logistic regression model:

$$\text{logit}(\widehat{\pi}(Age)) = -0.989 + 0.057 \cdot Age$$

- $\widehat{\beta}_0$: The estimated log-odds is -0.989 for someone who is 61.71 years (95% CI: -1.035, -0.944)
- $\widehat{\beta}_1$: The estimated increase in log-odds is 0.057 for every 1 year increase in age (95% CI: 0.051, 0.063).

- What does a log-odds even mean?? Or an increase in log-odds??
- We will need to calculate the **odds ratio** and its confidence interval
  - Then we will visualize the odds ratio

# Poll Everywhere Question 1

# We typically interpret our results using odds ratios

For our *fitted* simple logistic regression model with a continuous predictor

$$\text{logit}(\widehat{\pi}(X)) = \widehat{\beta}_0 + \widehat{\beta}_1 \cdot X$$

- How do we go from interpretations of $\widehat{\beta}_0$ and $\widehat{\beta}_1$ using log odds to odds ratios?

- We will need to take the exponential of our model:
  - $\exp(\widehat{\beta}_0)$: expected odds that $Y = 1$ when X is 0.
  - $\exp(\widehat{\beta}_1)$: expected odds ratio that $Y = 1$ for every 1 unit increase in X

- **Important distinction:**
  - We take the **inverse logit** to find our **predicted probability**
  - We take the **exponential** to interpret the **odds/odds ratios**

# Intro/Recap of Interpreting Fitted Model

- Interpret coefficients from fitted logistic regression model
  - **Goodness-of-fit of model should be assessed before summarizing findings** (have not covered yet)
  - In this lecture: assume model fits data well

- The interpretation of the coefficients involves two issues:
  - The functional relationship between the dependent variable and the independent variable (*link function*)
  - Unit of change for the independent variable

- We will learn the interpretation for
  - Binary independent variable
  - Categorical independent variable with multiple groups
    - We looked at this for our race and ethnicity variable
  - Continuous independent variable

# Learning Objectives

1. Interpret odds ratios from fitted simple logistic regression model for a continuous explanatory variable.
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3. Interpret odds ratios from fitted simple logistic regression model for a multi-level categorical explanatory variable.
4. Report the odds ratio using a table and/or a forest plot.

# Coefficient interpretation: Continuous Independent Variable

- For simplicity, we assume the linear relationship between logit and continuous variable $X$

- Again using simple logistic regression model to illustrate the interpretation of $\widehat{\beta}$ for a continuous variable $X$

$$\text{logit}(\widehat{\pi}(X)) = \widehat{\beta}_0 + \widehat{\beta}_1 \cdot X$$

- The estimated slope coefficient, $\widehat{\beta}_1$, is the **expected change in the log odds for 1 unit increase in** $X$
  - Additional attention should be paid to picking a meaningful units of change in $X$

# How do we get the odds ratio for X's coefficient?

For $\exp(\widehat{\beta}_1)$

- We compare $X = x$ and $X = x + 1$,
- So we have

$$\text{logit}(\widehat{\pi}(X = x)) = \widehat{\beta}_0 + \widehat{\beta}_1 \cdot x$$

and

$$\text{logit}(\widehat{\pi}(X = x + 1)) = \widehat{\beta}_0 + \widehat{\beta}_1 \cdot (x + 1)$$

- And...

$$\begin{aligned} &\text{logit}(\widehat{\pi}(X = x + 1)) - \text{logit}(\widehat{\pi}(X = x)) \\ &= \widehat{\beta}_0 + \widehat{\beta}_1 \cdot (x + 1) - \left[\widehat{\beta}_0 + \widehat{\beta}_1 \cdot x\right] \\ &= \widehat{\beta}_0 + \widehat{\beta}_1 \cdot x + \widehat{\beta}_1 - \widehat{\beta}_0 - \widehat{\beta}_1 \cdot x \\ &= \widehat{\beta}_1 \end{aligned}$$

- Thus,

$$\widehat{\beta}_1 = \text{logit}(\widehat{\pi}(X = x + 1)) - \text{logit}(\widehat{\pi}(X = x))$$

$$\widehat{\beta}_1 = \log\left(\frac{\widehat{\pi}(X = x + 1)}{1 - \widehat{\pi}(X = x + 1)}\right) - \log\left(\frac{\widehat{\pi}(X = x)}{1 - \widehat{\pi}(X = x)}\right)$$

$$\widehat{\beta}_1 = \log\left(\frac{\dfrac{\widehat{\pi}(X = x + 1)}{1 - \widehat{\pi}(X = x + 1)}}{\dfrac{\widehat{\pi}(X = x)}{1 - \widehat{\pi}(X = x)}}\right)$$

$$\exp\left[\widehat{\beta}_1\right] = \exp\left[\log\left(\frac{\widehat{\text{odds}}_{X=x+1}}{\widehat{\text{odds}}_{X=x}}\right)\right]$$

$$\exp\left[\widehat{\beta}_1\right] = \frac{\widehat{\text{odds}}_{X=x+1}}{\widehat{\text{odds}}_{X=x}}$$

# Example: Interpretation of Age Coefficient/OR

- $\widehat{\beta}_1$ is 0.057, suggesting that one year increase in age is associated with 0.057 increase in log odds of receiving a late stage breast cancer diagnosis

- $\exp\left(\widehat{\beta}_1\right)$ is 1.06, suggesting that one year increase in age is associated with 1.06 times the odds of receiving a late stage breast cancer diagnosis
  - Can also say: For every one year increase in age, the odds of late stage breast cancer diagnosis is 1.06 times.

- For continuous covariates in logistic regression model, it is helpful to subtract 1 from the odds ratio and multiply by 100 to obtain the percentage change in odds for 1-unit increase.
  - The estimated OR for age is 1.06, suggesting that a 1-year increase in age is associated with a 6% increase in the predicted odds of late stage diagnosis

# Example: Age and Late Stage Diagnosis (1/5)

## Odds ratio from logistic regression

Compute the estimate and 95% confidence interval for odds ratio for late stage breast cancer diagnosis for every 1 year increase in age.

**Needed steps:**

1. Fit the regression model
2. Transform the coefficients into odds ratios
3. Interpret the odds ratio

# Example: Age and Late Stage Diagnosis (2/5)

## Odds ratio from logistic regression

Compute the estimate and 95% confidence interval for odds ratio for late stage breast cancer diagnosis for every 1 year increase in age.

1. Fit the regression model

```r
bc_reg = bc %>%
  glm(formula = Late_stage_diag ~ Age_c, family = binomial)
```

# Example: Age and Late Stage Diagnosis (3/5)

**Odds ratio from logistic regression**

Compute the estimate and 95% confidence interval for odds ratio for late stage breast cancer diagnosis for every 1 year increase in age.

2. Transform the coefficients into odds ratios • Option 1: `tidy()`

```
tidy_bc_reg = tidy(bc_reg, conf.int=T, exponentiate = T)
tidy_bc_reg %>% gt() %>% tab_options(table.font.size = 35) %>%
  fmt_number(decimals = 3)
```

| term | estimate | std.error | statistic | p.value | conf.low | conf.high |
|---|---|---|---|---|---|---|
| (Intercept) | 0.372 | 0.023 | −42.637 | 0.000 | 0.355 | 0.389 |
| Age_c | 1.059 | 0.003 | 17.780 | 0.000 | 1.052 | 1.065 |

```
tidy_bc_reg$conf.low # I prefer tidy() bc now I can grab each component
```

```
[1] 0.3551931 1.0520321
```

# Example: Age and Late Stage Diagnosis (4/5)

**Odds ratio from logistic regression**

Compute the estimate and 95% confidence interval for odds ratio for late stage breast cancer diagnosis for every 1 year increase in age.

2. Transform the coefficients into odds ratios • Option 2: `logistic.display()`

```r
logistic.display(bc_reg) # Cannot grab each component in this
```

```
Logistic regression predicting Late_stage_diag : 1 vs 0

                   OR(95%CI)         P(Wald's test) P(LR-test)
Age_c (cont. var.) 1.06 (1.05,1.07)  < 0.001        < 0.001

Log-likelihood = -5754.8442
No. of observations = 10000
AIC value = 11513.6884
```

# Poll Everywhere Question 2

# Example: Age and Late Stage Diagnosis (5/5)

**Odds ratio from logistic regression**

Compute the estimate and 95% confidence interval for odds ratio for late stage breast cancer diagnosis for every 1 year increase in age.

3. Interpret the odds ratio

For every one year increase in age, there is an expected 5.86% increase in the odds of late stage breast cancer diagnosis (95% CI: 5.2%, 6.53%).

# Transformations of continuous variable to make more interpretable

- Sometimes a change in "1" unit may not be considered clinically interesting

- For example, a 1 year increase in age or a 1 mm Hg increase in systolic blood pressure may be too small for a meaningful change in log odds
  - Instead, we may be interested to find out the log odds change for a increase of 10 years in age or 10 mm Hg in systolic blood pressure

- On the other hand, if the range of $X$ is small (say 0-1), than a change in 1 unit of $X$ is too large to be meaningful

- We should be able to compute and interpret coefficients for a continuous independent covariate $x$ for an arbitrary change of "c" units in $x$

# Transformations of continuous variable to make more interpretable

- The estimated log odds ratio for a change of c units in x can be obtained from

$$\hat{g}\left(x+c\right)-\hat{g}\left(x\right)=c\hat{\beta}_1$$

  - $\widehat{OR}\left(c\right)=\exp\left(c\hat{\beta}_1\right)$

- The 95% CI for $\widehat{OR}(c)$ is:

$$\exp\left(c\hat{\beta}_1 \pm 1.96\cdot c\cdot SE_{\hat{\beta}_1}\right)$$

- The $c$ is chosen to be a clinically meaningful unit change in $x$
- The value of $c$ should be clearly specified in all tables and calculations
  - Because the estimated OR and the corresponding CI depends on the choice of $c$ value

# Example: 10 year increase in age and Late Stage Diagnosis

- What if we are interested in learning the OR corresponding to 10-year increase in age?

```
bc_reg = glm(Late_stage_diag ~ Age_c, data = bc, family = binomial)
tidy(bc_reg, conf.int=T) %>% gt() %>% tab_options(table.font.size = 35) %>%
  fmt_number(decimals = 3)
```

| term | estimate | std.error | statistic | p.value | conf.low | conf.high |
|---|---|---|---|---|---|---|
| (Intercept) | −0.989 | 0.023 | −42.637 | 0.000 | −1.035 | −0.944 |
| Age_c | 0.057 | 0.003 | 17.780 | 0.000 | 0.051 | 0.063 |

# Example: 10 year increase in age and Late Stage Diagnosis

- What if we are interested in learning the OR corresponding to 10-year increase in age?

$$\widehat{OR}(10) = \exp\left(10 \cdot \hat{\beta}_1\right) = \exp(0.56965) = 1.767$$

- The 95% CI for $\widehat{OR}(10)$ is:

$$\begin{aligned}\widehat{OR}(10) &= \exp\left(10 \cdot \hat{\beta}_1 \pm 1.96 \cdot 10 \cdot SE_{\hat{\beta}_1}\right) \\ &= \exp(10 \cdot 0.056965 \pm 1.96 \cdot 10 \cdot 0.003204) \\ &= (1.66,\ 1.88)\end{aligned}$$

# Example: 10 year increase in age and Late Stage Diagnosis

- What if we are interested in learning the OR corresponding to 10-year increase in age?

```
bc2 = bc %>% mutate(Age_c_10 = Age_c/10)
bc_reg_10 = glm(Late_stage_diag ~ Age_c_10, data = bc2, family = binomial)
tidy(bc_reg_10, conf.int=T, exponentiate = T) %>% gt() %>% tab_options(table.font.size = 35
  fmt_number(decimals = 3)
```

| term | estimate | std.error | statistic | p.value | conf.low | conf.high |
|---|---|---|---|---|---|---|
| (Intercept) | 0.372 | 0.023 | −42.637 | 0.000 | 0.355 | 0.389 |
| Age_c_10 | 1.768 | 0.032 | 17.780 | 0.000 | 1.661 | 1.883 |

# Last Note About Continuous Independent Variable

- Notice that the logistic regression model suggests that logit is linear in the covariate
- The model implies the additional risk of late stage breast cancer diagnosis for a 40 year-old compared to a 30 year-old is the same as the additional risk of late stage breast cancer diagnosis for a 60 year-old compared to a 50-year-old
- This assumption may not be realistic
- To address this, we may consider using higher order terms (e.g., $x^2$, $x^3$,...) or other nonlinear transformation(e.g., $log(x)$)
- Categorize the continuous variable may be another option

# How do we get the odds for the intercept?

For $\exp(\widehat{\beta}_0)$

- When $X = 0$, we have

$$\text{logit}(\widehat{\pi}(X=0)) = \widehat{\beta}_0$$

- Thus,

$$\begin{aligned}
\widehat{\beta}_0 &= \text{logit}(\widehat{\pi}(X)) \\
\exp\left[\widehat{\beta}_0\right] &= \exp\left[\text{logit}(\widehat{\pi}(X))\right] \\
\exp\left[\widehat{\beta}_0\right] &= \exp\left[\log\left(\frac{\widehat{\pi}(X)}{1-\widehat{\pi}(X)}\right)\right] \\
\exp\left[\widehat{\beta}_0\right] &= \frac{\widehat{\pi}(X)}{1-\widehat{\pi}(X)}
\end{aligned}$$

# Learning Objectives

1. Interpret odds ratios from fitted simple logistic regression model for a continuous explanatory variable.
2. Interpret odds ratios from fitted simple logistic regression model for a binary explanatory variable.
3. Interpret odds ratios from fitted simple logistic regression model for a multi-level categorical explanatory variable.
4. Report the odds ratio using a table and/or a forest plot.

# Coefficient Interpretation: Binary Independent Variable

- Independent variable $X$ is a binary variable ($X$ can take values: 0 or 1)

- We are fitting the simple logistic regression model:

$$\text{logit}\left(\pi(X)\right) = \beta_0 + \beta_1 \cdot I(X = 1)$$

- The logit difference is $\beta_1$ for binary independent variable
  - $\beta_1$ represents the change/difference in the logit for $X = 1$ vs. $X = 0$

- It will be much easier to understand if we can interpret the coefficient using odds ratio (OR)

# Binary: How do we interpret the coefficient? (I)

- For individuals with $X = 0$:

$$\text{logit}\left(\pi(X=0)\right) = \beta_0 + \beta_1 \times (0) = \beta_0$$

- For individuals with $X = 1$:

$$\text{logit}\left(\pi(X=1)\right) = \beta_0 + \beta_1 \times (1) = \beta_0 + \beta_1$$

- To solve for $\beta_1$, we take the difference of the logits:

$$\text{logit}\left(\pi(X=1)\right) - \text{logit}\left(\pi(X=0)\right) = (\beta_0 + \beta_1) - (\beta_0) = \beta_1$$

# Binary: How do we interpret the coefficient? (II)

$$\text{logit}\left(\pi(X=1)\right) - \text{logit}\left(\pi(X=0)\right) = (\beta_0 + \beta_1) - (\beta_0) = \beta_1$$

$$\beta_1 = logit\left(\pi(X=1)\right) \; - logit\left(\pi\left(X=0\right)\right)$$

$$\beta_1 = log\left(\frac{\pi(X=1)}{1-\pi(X=1)}\right) - log\left(\frac{\pi\left(X=0\right)}{1-\pi\left(X=0\right)}\right)$$

$$\beta_1 = \log\left(\frac{\frac{\pi(X=1)}{1-\pi(X=1)}}{\frac{\pi(X=0)}{1-\pi(X=0)}}\right)$$

$$\exp\left(\beta_1\right) = \frac{\frac{\pi(X=1)}{1-\pi(X=1)}}{\frac{\pi(X=0)}{1-\pi(X=0)}}$$

# Review of Odds Ratio

- Odds for a subject with $X = 1$:

$$\text{odds}_1 = \frac{\pi(X=1)}{1-\pi(X=1)}$$

- Odds for a subject with $X = 0$:

$$\text{odds}_0 = \frac{\pi(X=0)}{1-\pi(X=0)}$$

- Odds Ratio for $X = 1$ vs. $X = 0$:

$$OR = \frac{\dfrac{\pi(X=1)}{1-\pi(X=1)}}{\dfrac{\pi(X=0)}{1-\pi(X=0)}}$$

# How does this relate to a 2x2 table?

- 2x2 table with the respective logistic functions in each cell

| Outcome Variable (Y) | Independent Variable (X) | |
|---|---|---|
| | $X = 1$ | $X = 0$ |
| $Y = 1$ | $\pi(1) = \dfrac{\exp(\beta_0 + \beta_1)}{1 + \exp(\beta_0 + \beta_1)}$ | $\pi(0) = \dfrac{\exp(\beta_0)}{1 + \exp(\beta_0)}$ |
| $Y = 0$ | $1 - \pi(1) = \dfrac{1}{1 + \exp(\beta_0 + \beta_1)}$ | $1 - \pi(0) = \dfrac{1}{1 + \exp(\beta_0)}$ |
| Total | 1.0 | 1.0 |

**Recall:**

$\pi(1) = \pi(X = 1)$
$= P(Y = 1|X = 1)$

$\pi(0) = \pi(X = 0)$
$= P(Y = 1|X = 0)$

# Poll Everywhere Question 3

# Example: Binary age and Late Stage Diagnosis (1/4)

**Odds ratio from logistic regression**

What is the odds ratio of late stage breast cancer diagnosis for older individuals (>65 years old) compared to younger individuals (≤65 years old)?

- Two options to calculate this value:
  - Option 1: Calculate $\widehat{OR}$ from 2x2 contingency table
    - Try at home: Refer to Lesson 3 for this process
  - Option 2: Calculate $\widehat{OR}$ from logistic regression

**Needed steps for Option 2:**

1. Fit the regression model
2. Transform the coefficients into odds ratios
3. Interpret the odds ratio

# Example: Binary age and Late Stage Diagnosis (2/4)

**Odds ratio from logistic regression**

What is the odds ratio of late stage breast cancer diagnosis for older individuals (>65 years old) compared to younger individuals (≤65 years old)?

1. Fit the regression model

```
bc3 = bc %>% mutate(Age_binary = ifelse(Age > 65, 1, 0))
age_bin_glm = bc3 %>% glm(formula = Late_stage_diag ~ Age_binary, family = binomial)
```

# Example: Binary age and Late Stage Diagnosis (3/4)

**Odds ratio from logistic regression**

What is the odds ratio of late stage breast cancer diagnosis for older individuals (>65 years old) compared to younger individuals (≤65 years old)?

2. Transform the coefficients into odds ratios

```
age_bin_tidy = tidy(age_bin_glm, conf.int=T, exponentiate = T)
age_bin_tidy %>% gt() %>%
  tab_options(table.font.size = 35) %>%
  fmt_number(decimals = 3)
```

| term | estimate | std.error | statistic | p.value | conf.low | conf.high |
|---|---|---|---|---|---|---|
| (Intercept) | 0.297 | 0.031 | −39.608 | 0.000 | 0.280 | 0.315 |
| Age_binary | 1.875 | 0.045 | 13.928 | 0.000 | 1.716 | 2.048 |

# Poll Everywhere Question 4

# Example: Binary age and Late Stage Diagnosis (4/4)

**Odds ratio from logistic regression**

What is the odds ratio of late stage breast cancer diagnosis for older individuals (>65 years old) compared to younger individuals (≤65 years old)?

3. Interpret the odds ratio

The estimated odds of late stage breast cancer among individuals over 65 years old is 1.87 (95% CI: (1.72, 2.05)) times that of individuals 65 years or younger.

# Learning Objectives

1. Interpret odds ratios from fitted simple logistic regression model for a continuous explanatory variable.
2. Interpret odds ratios from fitted simple logistic regression model for a binary explanatory variable.
3. Interpret odds ratios from fitted simple logistic regression model for a multi-level categorical explanatory variable.
4. Report the odds ratio using a table and/or a forest plot.

# Coefficient Interpretation: Multi-group Categorical Variable

- Independent variable $X$ is a multi-level categorical variable

- Let's say $X$ takes values: a, b, c, or d

- We are fitting the simple logistic regression model:

$$\text{logit}\left(\pi(X)\right) = \beta_0 + \beta_1 \cdot I(X = b) + \beta_2 \cdot I(X = c) + \beta_3 \cdot I(X = d)$$

  - Where $a$ is our reference group

- $\beta_1$ represents the change/difference in the log-odds for $X = b$ vs. $X = a$

# Coefficient Interpretation: Multi-group Categorical Variable

We are fitting the simple logistic regression model with reference group $a$:

$$\text{logit}\left(\pi(X)\right) = \beta_0 + \beta_1 \cdot I(X=b) + \beta_2 \cdot I(X=c) + \beta_3 \cdot I(X=d)$$

- $\beta_0$: the log-odds of event $Y=1$ for group $a$
- $\beta_1$: the difference in log-odds of event $Y=1$ comparing group $b$ to group $a$
- $\beta_2$: the difference in log-odds of event $Y=1$ comparing group $c$ to group $a$
- $\beta_3$: the difference in log-odds of event $Y=1$ comparing group $d$ to group $a$

# Multi-level categorical: How do we interpret the coefficient? (II)

$$\operatorname{logit}(\pi(X=c)) - \operatorname{logit}(\pi(X=a)) = (\beta_0 + \beta_1 \cdot 0 + \beta_2 \cdot 1 + \beta_3 \cdot 0) - (\beta_0 + \beta_1 \cdot 0 + \beta_2 \cdot 0 + \beta_3 \cdot 0) = \beta_2$$

$$\beta_2 = \operatorname{logit}(\pi(X=c)) - \operatorname{logit}(\pi(X=a))$$

$$\beta_2 = \log\left(\frac{\pi(X=c)}{1-\pi(X=c)}\right) - \log\left(\frac{\pi(X=a)}{1-\pi(X=a)}\right)$$

$$\beta_2 = \log\left(\frac{\frac{\pi(X=c)}{1-\pi(X=c)}}{\frac{\pi(X=a)}{1-\pi(X=a)}}\right)$$

$$\exp(\beta_2) = \frac{\frac{\pi(X=c)}{1-\pi(X=c)}}{\frac{\pi(X=a)}{1-\pi(X=a)}}$$

# Coefficient Interpretation: Multi-group Categorical Variable

We are fitting the simple logistic regression model with reference group $a$:

$$\text{logit}\left(\pi(X)\right) = \beta_0 + \beta_1 \cdot I(X = b) + \beta_2 \cdot I(X = c) + \beta_3 \cdot I(X = d)$$

- $\exp\left(\beta_0\right)$: the odds of event $Y = 1$ for group $a$
- $\exp\left(\beta_1\right)$: the odds of event $Y = 1$ for group $b$ is $\exp\left(\beta_1\right)$ times the odds of event $Y = 1$ for group $a$
- $\exp\left(\beta_2\right)$: the odds of event $Y = 1$ for group $c$ is $\exp\left(\beta_2\right)$ times the odds of event $Y = 1$ for group $a$
- $\exp\left(\beta_3\right)$: the odds of event $Y = 1$ for group $d$ is $\exp\left(\beta_3\right)$ times the odds of event $Y = 1$ for group $a$

- Remember, as soon as we fit the regression model, we talk about the “expected odds” or “estimated odds”

# How do we pick the reference group?

- The choice can be more apparent for multi-group categorical independent variables within studies

- For example, if we want to evaluate the association between clinical response and four treatments.
  - The treatment variable has 4 categories: “active treatment A”, “active treatment B”, “active treatment C” and “Placebo treatment”
  - The investigator is interested in comparing each of the three active treatment with the placebo treatment
  - Then the placebo treatment should be picked as the reference group

# Example: Late stage diagnosis and race and ethnicity

- We chose Non-Hispanic White individuals as reference group

- Underlying health disparities linked to racism in healthcare and in clinical studies

- There is evidence that white individuals receive a certain standard of care that is not paralleled for POC (Mateo and Williams (2021))

| | Breast Cancer Diagnosis | | |
|---|---|---|---|
| **Race/Ethnicity** | **Early Stage** | **Late Stage** | **Total** |
| **Non-Hispanic White** | 5,321 | 1,980 | 7,301 |
| **Non-Hispanic Black** | 683 | 357 | 1,040 |
| **Non-Hispanic Asian/Pacific Islander** | 556 | 234 | 790 |
| **Hispanic-Latinx** | 575 | 271 | 846 |
| **Non-Hispanic American Indian/Alaska Native** | 17 | 6 | 23 |
| **Total** | 7,152 | 2,848 | 10,000 |

- Something else to consider: if your reference group does not have a lot of individuals, then you may not see any statistical significance!

# Example: Late stage diagnosis and race and ethnicity

**Odds ratio from logistic regression**

What is the odds ratio of late stage breast cancer diagnosis for Non-Hispanic Asian/Pacific Islander individuals compared to Non-Hispanic White individuals?

**Needed steps:**

1. Fit the regression model
2. Transform the coefficients into odds ratios
3. Interpret the odds ratio

# Example: Late stage diagnosis and race and ethnicity

**Odds ratio from logistic regression**

What is the odds ratio of late stage breast cancer diagnosis for Non-Hispanic Asian/Pacific Islander individuals compared to Non-Hispanic White individuals?

1. Fit the regression model

```r
RE_glm = bc %>%
  glm(formula = Late_stage_diag ~ Race_Ethnicity, family = binomial)
```

# Example: Late stage diagnosis and race and ethnicity

**Odds ratio from logistic regression**

What is the odds ratio of late stage breast cancer diagnosis for Non-Hispanic Asian/Pacific Islander individuals compared to Non-Hispanic White individuals?

2. Transform the coefficients into odds ratios

```
RE_tidy = tidy(RE_glm, conf.int=T, exponentiate = T)
RE_tidy %>% gt() %>%
  tab_options(table.font.size = 35) %>%
  fmt_number(decimals = 3)
```

| term | estimate | std.error | statistic | p.value | conf.low | conf.high |
|---|---|---|---|---|---|---|
| (Intercept) | 0.372 | 0.026 | −37.553 | 0.000 | 0.353 | 0.392 |
| Race_EthnicityHispanic-Latino | 0.968 | 0.082 | −0.398 | 0.691 | 0.822 | 1.135 |
| Race_EthnicityNH American Indian/Alaskan Native | 0.948 | 0.476 | −0.111 | 0.911 | 0.342 | 2.287 |
| Race_EthnicityNH Asian/Pacific Islander | 1.131 | 0.082 | 1.497 | 0.134 | 0.961 | 1.327 |
| Race_EthnicityNH Black | 1.405 | 0.070 | 4.826 | 0.000 | 1.223 | 1.611 |

# Example: Late stage diagnosis and race and ethnicity

**Odds ratio from logistic regression**

What is the odds ratio of late stage breast cancer diagnosis for Non-Hispanic Asian/Pacific Islander individuals compared to Non-Hispanic White individuals?

3. Interpret the odds ratio

The estimated odds of late stage breast cancer among Non-Hispanic Asian/Pacific Islander individuals is 1.13 (95% CI: (0.96, 1.33)) times that of Non-Hispanic White individuals.

# What if you want to compare other groups?

- What if we want to estimate OR comparing Non-Hispanic Asian Pacific Islander to Non-Hispanic Black individuals?
- Option 1: Change reference group and refit the model (maybe the easiest option)
- Option 2: Estimate OR using fitted coefficients ($\widehat{\beta}$'s) in the current model:

$$\begin{aligned}
\log\left(OR(\text{NH API}, \text{NH B})\right) &= \text{logit}\left(\pi\left(X = \text{NH API}\right)\right) - \text{logit}\left(\pi\left(X = \text{NH B}\right)\right) \\
&= [\beta_0 + \beta_3 \cdot 1] - [\beta_0 + \beta_4 \cdot 1] \\
\log\left(\widehat{OR}(\text{NH API}, \text{NH B})\right) &= \widehat{\beta}_3 - \widehat{\beta}_4 \\
\widehat{OR}(\text{NH API}, \text{NH B}) &= \exp\left(\widehat{\beta}_3 - \widehat{\beta}_4\right)
\end{aligned}$$

# Reference: Using `estimable()` in Option 2

- We could use `estimable()` to calculate this linear combination of coefficients (see BSTA 512 Lesson)

```r
library(gmodels)
RE_glm %>% estimable(
                   c("(Intercept)"                                  = 0,   # beta0
                     "Race_EthnicityHispanic-Latino"                = 0,   # beta1
                     "Race_EthnicityNH American Indian/Alaskan Native" = 0,   # beta2
                     "Race_EthnicityNH Asian/Pacific Islander"      = 1,   # beta3
                     "Race_EthnicityNH Black"                       = -1), # beta4
                   conf.int = 0.95) %>%
  exp(.)
```

```
              Estimate Std. Error X^2 value       DF Pr(>|X^2|)  Lower.CI
(0 0 0 1 -1) 0.8051811   1.107021  93.89366 2.718282   1.033623 0.6580966
              Upper.CI
(0 0 0 1 -1) 0.9851389
```

# Poll Everywhere Question 5

# What if you want to compare other groups? Option 1

```r
bc3 = bc %>%
  mutate(Race_Ethnicity = relevel(Race_Ethnicity, ref = "NH Black"))
RE_glm2 = glm(Late_stage_diag ~ Race_Ethnicity, data = bc3,
               family = binomial)
tidy(RE_glm2, conf.int=T, exponentiate = T) %>% gt() %>%
  tab_options(table.font.size = 38) %>%
  fmt_number(decimals = 3)
```

| term | estimate | std.error | statistic | p.value | conf.low | conf.high |
|---|---|---|---|---|---|---|
| (Intercept) | 0.523 | 0.065 | −9.934 | 0.000 | 0.459 | 0.594 |
| Race_EthnicityNH White | 0.712 | 0.070 | −4.826 | 0.000 | 0.621 | 0.818 |
| Race_EthnicityHispanic-Latino | 0.689 | 0.102 | −3.664 | 0.000 | 0.564 | 0.840 |
| Race_EthnicityNH American Indian/Alaskan Native | 0.675 | 0.479 | −0.819 | 0.413 | 0.242 | 1.641 |
| Race_EthnicityNH Asian/Pacific Islander | 0.805 | 0.102 | −2.131 | 0.033 | 0.659 | 0.982 |

# Learning Objectives

1. Interpret odds ratios from fitted simple logistic regression model for a continuous explanatory variable.
2. Interpret odds ratios from fitted simple logistic regression model for a binary explanatory variable.
3. Interpret odds ratios from fitted simple logistic regression model for a multi-level categorical explanatory variable.

4. Report the odds ratio using a table and/or a forest plot.

# How to present odds ratios: Table

- `tbl_regression()` in the `gtsummary` package is helpful for presenting the odds ratios in a clean way

```
library(gtsummary)
tbl_regression(RE_glm, exponentiate = TRUE) %>%
  as_gt() %>% # allows us to use tab_options()
  tab_options(table.font.size = 38)
```

| Characteristic | OR[1] | 95% CI[1] | p-value |
|---|---|---|---|
| Race_Ethnicity | | | |
| NH White | — | — | |
| Hispanic-Latino | 0.97 | 0.82, 1.14 | 0.7 |
| NH American Indian/Alaskan Native | 0.95 | 0.34, 2.29 | >0.9 |
| NH Asian/Pacific Islander | 1.13 | 0.96, 1.33 | 0.13 |
| NH Black | 1.40 | 1.22, 1.61 | <0.001 |

[1] OR = Odds Ratio, CI = Confidence Interval

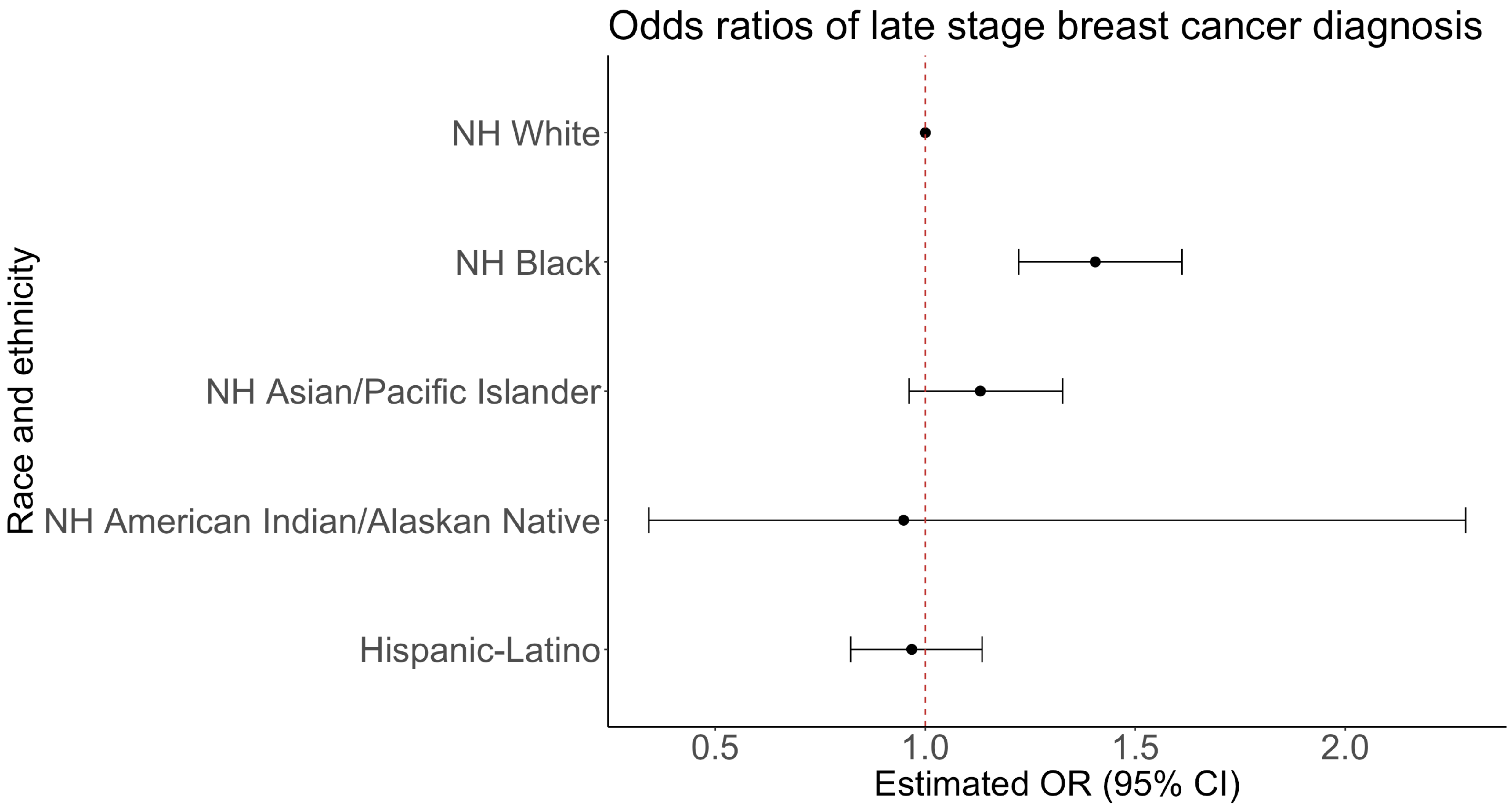
# How to present odds ratios: Forest Plot Setup

```r
library(broom.helpers)
RE_tidy = tidy_and_attach(RE_glm, conf.int=T, exponentiate = T) %>%
  tidy_remove_intercept() %>%
  tidy_add_reference_rows() %>% tidy_add_estimate_to_reference_rows() %>%
  tidy_add_term_labels()
glimpse(RE_tidy)
```

```
Rows: 5
Columns: 16
$ term           <chr> "Race_EthnicityNH White", "Race_EthnicityHispanic-Latin…
$ variable       <chr> "Race_Ethnicity", "Race_Ethnicity", "Race_Ethnicity", "…
$ var_label      <chr> "Race_Ethnicity", "Race_Ethnicity", "Race_Ethnicity", "…
$ var_class      <chr> "factor", "factor", "factor", "factor", "factor"
$ var_type       <chr> "categorical", "categorical", "categorical", "categoric…
$ var_nlevels    <int> 5, 5, 5, 5, 5
$ contrasts      <chr> "contr.treatment", "contr.treatment", "contr.treatment"…
$ contrasts_type <chr> "treatment", "treatment", "treatment", "treatment", "tr…
$ reference_row  <lgl> TRUE, FALSE, FALSE, FALSE, FALSE
$ label          <chr> "NH White", "Hispanic-Latino", "NH American Indian/Alas…
$ estimate       <dbl> 1.0000000, 0.9678002, 0.9484848, 1.1310170, 1.4046741
$ std.error      <dbl> NA, 0.08224948, 0.47558680, 0.08224988, 0.07041472
$ statistic      <dbl> NA, -0.3979312, -0.1112089, 1.4968682, 4.8257715
$ p.value        <dbl> NA, 6.906809e-01, 9.114507e-01, 1.344276e-01, 1.394623e…
$ conf.low       <dbl> NA, 0.8223138, 0.3417844, 0.9612074, 1.2226824
$ conf.high      <dbl> NA, 1.135332, 2.286596, 1.327092, 1.611466
```

# How to present odds ratios: Forest Plot

```r
ggplot(data=RE_tidy, aes(y=label, x=estimate, xmin=conf.low, xmax=conf.high)) +
  geom_point(size = 3) +  geom_errorbarh(height=.2) +
  geom_vline(xintercept=1, color='#C2352F', linetype='dashed', alpha=1) +
  theme_classic() +
  labs(x = "Estimated OR (95% CI)", y = "Race and ethnicity",
       title = "Odds ratios of late stage breast cancer diagnosis") +
  theme(axis.title = element_text(size = 25), axis.text = element_text(size = 25), title =
```



# References

Mateo, Camila M., and David R. Williams. 2021. "Racism: A Fundamental Driver of Racial Disparities in Health-Care Quality." *Nature Reviews Disease Primers* 7 (1): 1–2. https://doi.org/10.1038/s41572-021-00258-1.

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