Lesson 15: Other types of categorical regression

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Learning Objectives

1. Review Generalized Linear Models and how we can branch to other types of regression.
2. Identify outcome, examples, population model, and interpretations for different generalized linear models.

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Review: Generalized Linear Models (GLMs)

Generalized Linear Models

Random component

- Identify the response variable Y
- Specify a suitable (presumably) distribution for it

Systematic component

- Specify the explanatory variable(s) for the model

X's & β's

will mean something different

Link function

- Specify a functional form of $E(Y)$ that is related to the explanatory variables through a prediction equation in linear form

if $Y \sim \text{binom}(p)$

$Y \neq X\beta$

$\text{logit}(P(Y=1))$

GLM: Random Component

- The random component specifies the response variable Y and selects a probability distribution for it

- Basically, we are just identifying the distribution for our outcome
  - If Y is binary: assumes a binomial distribution of Y
  - If Y is count: assumes Poisson or negative binomial distribution of Y
  - If Y is continuous: assumea Normal distribution of Y

GLM: Systematic Component

- The systematic component specifies the explanatory variables, which enter linearly as predictors

cont Y

Y

$$\beta_0 + \beta_1 X_1 + \ldots + \beta_k X_k$$

- Above equation includes:
  - Centered variables ✓ $X - \mu_x$
  - Interactions ✓ $X_1 \times X_2$
  - Transformations of variables (like squares) ✓ $(X - \mu_x)^2$ or X^2

- Systematic component is the **same** as what we learned in Linear Models

GLM: Link Function

- If $\mu = E(Y)$, then the link function specifies a function $g(.)$ that relates μ to the linear predictors as:

$$g(\mu) = \beta_0 + \beta_1 X_1 + \ldots + \beta_k X_k$$

  - $g(\mu)$ is the transformation we make to $E(Y)$ (aka μ) so that the linear predictors (right side of equation) can be linked to the outcome
- The link function connects the random component with the systematic component
- Can also think of this as:

$$\mu = g^{-1}(\beta_0 + \beta_1 X_1 + \ldots + \beta_k X_k)$$

★ It's basically like saying $g(\mu)$ **IS** $\text{logit}(\mu)$ and thus

$g(\mu) = \text{logit}(\mu) = \text{logit}(P(Y=1|X))$

$$\underbrace{\text{logit}(\mu)}_{\text{link (informed by random)}} = \underbrace{\beta_0 + \beta_1 X_1 + \ldots + \beta_k X_k}_{\text{sys}}$$

link fn IS logit (μ)

GLM: Link Function

numeric data / cont

Dist'n of Y	Typical uses	Link name	Link function	Common name
512 Normal	Linear-response data	Identity	$g(\mu) = \mu$	Linear regression
Bernoulli / Binomial	outcome of single yes/no occurrence	Logit	$g(\mu) = \text{logit}(\mu)$	Logistic regression
Poisson	count of occurrences in fixed amount of time/space or rate	Log	$g(\mu) = \log(\mu)$	Poisson regression
Bernoulli / Binomial	outcome of single yes/no occurrence	Log	$g(\mu) = \log(\mu)$	Log-binomial regression
Multinomial	outcome of single occurence with K > 2 options, *nominal*	Logit	$g(\mu) = \text{logit}(\mu)$	Multinomial logistic regression
Multinomial	outcome of single occurence with K > 2 options, *ordinal*	Logit	$g(\mu) = \text{logit}(\mu)$	Ordinal logistic regression

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What is the purpose of a link function?

X's

To transform the predictors/covaraites to a linear scale 0%

To make sure the outcome follows a normal distribution 0%

To make a transformation of the outcome so we can map our predictors to it ✓ 0%

To minimize the residual sum of squares. 0%

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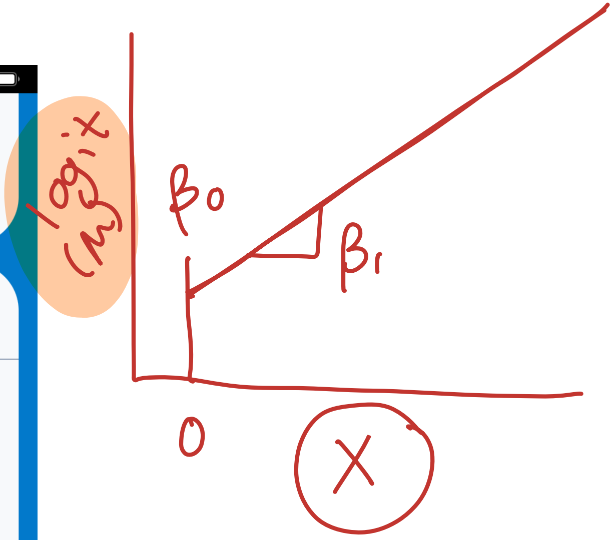

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2. Identify outcome, examples, population model, and interpretations for different generalized linear models.

Linear regression

- **Outcome type:** continuous , numeric
- **Example outcomes:**
  - Height ✓
  - IAT score ✓
  - Heart rate ✓

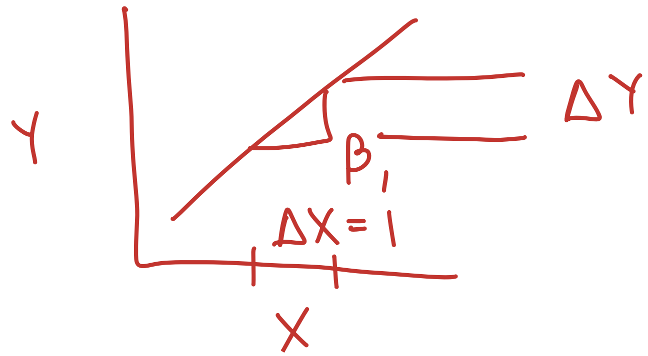
normal distribution

$Y \sim N(\mu, \sigma^2)$

- **Population model**

$$E(Y \mid X) = \mu = \beta_0 + \beta_1 X$$

- **Interpretations** of coeff. β_1
  - The change in average Y for every 1 unit increase in X



Linear regression: Process for data analysis

Model Selection

- Building a model
- Selecting variables
- Prediction vs interpretation
- Comparing potential models

Model Fitting

- Find best fit line
- Using OLS in this class
- Parameter estimation
- Categorical covariates
- Interactions

as X's

Model Evaluation

- Evaluation of model fit
- Testing model assumptions
- Residuals ✓
- Transformations
- Influential points ✓
- Multicollinearity ✓

Model Use (Inference)

- Inference for coefficients
- Hypothesis testing for coefficients
- Inference for expected Y given X
- Prediction of new Y given X

Logistic regression

- **Outcome type:** binary, yes or no
- **Example outcomes:**
  - Food insecurity ✓
  - Disease diagnosis for patient ✓
  - Fracture ✓

$Y \sim \text{binom}(p)$

- **Population model**

$$\text{logit}(\mu) = \text{logit}(\pi(X)) = \beta_0 + \beta_1 X$$

- **Interpretations** of β_1
  - The log-odds ratio for every 1 unit increase in X
  - $\exp(\beta_1)$ is odds ratio for every 1 unit increase in X

we can get risk ratio from this!

Logistic regression: Process for data analysis

Model Selection

- Build a model
- Select variables
- Prediction vs association
- Comparing potential models

Model Fitting

- Find model that maximizes likelihood function
- Parameter estimation (MLEs)
- Categorical covariates
- Interactions

Model Evaluation

- Evaluation of model fit
- Check model assumptions
- Transformations
- Influential points
- Multicollinearity
- Overdispersion

Model Use (Inference)

- Inference for odds ratios
- Hypothesis testing for odds ratios
- Inference for expected π given X
- Prediction of new Y given X

Log-binomial Regression

- Outcome type: binary, yes or no
- Example outcomes:
  - Food insecurity
  - Disease diagnosis for patient
  - Fracture

$Y \sim \text{binom}(p)$

prob can be > 1

$\mu \in [0, \infty)$

- Population model

$$\log(\mu) = \log(\pi(X)) = \beta_0 + \beta_1 X$$

- Interpretations of β_1
  - We have log of probability on the left
  - So exponential of our coefficients will be **risk ratio**

$\exp(\beta_1)$: risk ratio for every 1 unit inc in X

Poisson Regression

- **Outcome type:** Counts or rates

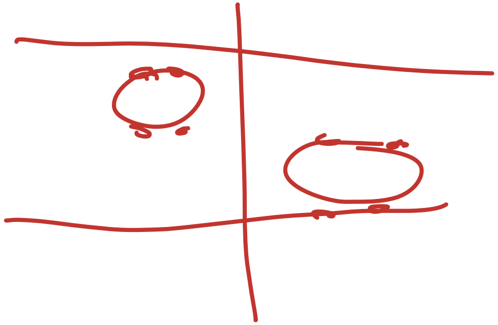
- **Example outcomes:**
  - Number of children in household
  - Number of hospital admissions
  - Rate of incidence for COVID in US counties

- **Population model**

mean
rate

$$\log(\mu) = \log(\lambda) = \beta_0 + \beta_1 X$$

- **Interpretations**
  - The count (or rate) ratio for every 1 unit increase in X



Count cars in 10 min
OR get rate cars/min

Multinomial logistic regression

- **Outcome type:** multi-level categorical, no inherent order

binary $\text{logit}(\pi) = \log\left(\frac{\pi}{1-\pi}\right)$ π yes π no

$\log\left(\frac{\mu_2}{\mu_1}\right) = \log\left(\frac{\pi_2}{\pi_1}\right)$

- **Example outcomes:**
  - Blood type —
  - US region (from WBNS)
  - Primary site of lung cancer (upper lobe, lower lobe, overlapped, etc.)
- We have additional restriction that the multiple group probabilities sum to 1

$P_{AB} + P_B + P_A + P_0 = 1$

- **Population models** 3 grp

$$\log\left(\frac{\mu_{\text{group 2}}}{\mu_{\text{group 1}}}\right) = \beta_0 + \beta_1 X$$

$$\log\left(\frac{\mu_{\text{group 3}}}{\mu_{\text{group 1}}}\right) = \beta_2 + \beta_3 X$$

$\log\left(\frac{\mu_{gp3}}{\mu_{gp2}}\right) = \beta_4 + \beta_5 X$

- **Interpretations**
  - Basically fitting two binary logistic regressions at same time!
  - β_1 First equation: how a one unit change in X changes the log-odds of going from group 1 to group 2
  - β_3 Second equation: how a one unit change in X changes the log-odds of going from group 1 to group 3

$\exp \beta_3 =$

Ordinal logistic regression

- Outcome type: multi-level categorical, with inherent order
- Example outcomes:
  - Satisfaction level (likert scale)
  - Pain level
  - Stages of cancer

logit(P(Y≤1))

P(Y>1)=
1−P(Y≤1)

- Population models , with levels $k = 1, 2, 3, \ldots, K$

$$\log\left(\frac{P(Y \leq 1)}{P(Y > 1)}\right) = \beta_0 + \beta_1 X$$

$$\log\left(\frac{P(Y \leq k)}{P(Y > k)}\right) = \beta_0 + \beta_1 X$$

- When these variables are predictors, we are pretty lenient about treating them as continuous
  - We must be VERY STRICT when they are outcomes
  - They do not meet the assumptions we place on continuous outcomes in linear regression!!
- We have additional restriction that the multiple group probabilities sum to 1

- Interpretations
  - Basically fitting K binary logistic regressions at same time!
  - First equation: how a one unit change in X changes the log-odds of going from group 1 to any other group
  - Second equation: how a one unit change in X changes the log-odds of going from group 1 or 2 to group 3 or above

Resources

Linear regression resources

- 512/612 class site!!
- Online textbook by Dr. Nahhas

Logistic regression resources

- Online textbook by Dr. Nahhas

Log-binomial Regression resources

- Online textbook by Dr. Nahhas
- Article on `logbin` package that is used to fit log-binomial regression

Ordinal logistic regression resources

- Online textbook by Dr. Nahhas
- Online textbook by Dr. Werth with data and R script

Poisson Regression resources

- PennState 504 website
- Online textbook by Dr. Nahhas
- YouTube video on R tutorial for Poisson Regression
  - Dr. Fogerty is a professor in Political Science, so just beware they may not have formal statistical training
- Guided R tutorial page on Poisson regression
- Online textbook by Dr. Werth
  - Social scientist, so just beware they may not have formal statistical training

Multinomial logistic regression resources

- YouTube video on R tutorial for Poisson Regression
  - Again, Dr. Fogerty is a professor in Political Science
- R-bloggers post with guided R code
- Online textbook by Dr. Werth with data and R script

Even more regressions...

Dist'n of Y	Typical uses	Link name	Link function	Common name
Bernoulli / Binomial	outcome of single yes/no occurrence	Probit	$g(\mu) = \Phi^{-1}(\mu)$	Probit regression
Bernoulli / Binomial	outcome of single yes/no occurrence	Complementary log-log	$g(\mu) = \log(-\log(1-\mu))$	Complementary log-log regression
Multinomial	outcome of single occurence with K > 2 options, *nominal*	Probit	$g(\mu) = \Phi^{-1}(\mu)$	Multinomial probit regression
Multinomial	outcome of single occurence with K > 2 options, *ordinal*	Probit	$g(\mu) = \Phi^{-1}(\mu)$	Ordered probit regression

More regression resources

- Probit regression
- Complementary log-log
- Multinomial probit
- Ordered probit

General resources

- Dr. Fogerty's YouTube series
- Dr. Werth's Categorical Book
- Dr. Nahhas' Book
- The Epidemiologist R Handbook
  - Analysis AND R work