Pre Chapter 24: Calculus Review

Meike Niederhausen and Nicky Wakim

2024-11-04

Learning Objectives

1. Find derivatives of continuous functions with one variable
2. Find antiderivatives and integrals of functions with one variable

Where are we?

Basics of probability

- Outcomes and events
- Sample space
- Probability axioms
- Probability properties
- Counting
- Independence
- Conditional probability
- Bayes' Theorem
- Random Variables

Probability for discrete random variables

- Functions: pmfs/CDFs
- Important distributions
- Joint distributions
- Expected values and variance

Probability for continuous random variables

- Calculus
- Functions: pdfs/CDFs
- Important distributions
- Joint distributions
- Expected values and variance

Advanced probability

- Central limit theorem
- Functions: moment generating functions

Differentiation

Find the *derivative* of the following function

Example 1.1

$f(x) = 2$

Derivative of a constant

$$\frac{d}{dx}c = 0$$

Find the *derivative* of the following function

Example 1.2

$f(x) = 2x$

Find the *derivative* of the following function

Example 1.3

$f(x) = 2x + 2$

Find the *derivative* of the following function

Example 1.4

$f(x) = x^2$

Derivative of x to a constant

$$\frac{d}{dx}x^n = nx^{n-1}$$

Find the *derivative* of the following function

Example 1.5

$f(x) = 3\sqrt{x} + \frac{2}{x} + 5$

Find the *derivative* of the following function

Example 1.6

$f(x) = e^x$

Derivative of exponential function

$$\frac{d}{dx} e^x = e^x$$

Find the *derivative* of the following function

Example 1.7

$f(x) = \ln(x)$

Derivative of logarithm

$$\frac{d}{dx} ln(x) = \frac{1}{x}$$

Find the *derivative* of the following function

Example 1.8

$f(x) = x^2 e^x$

Product Rule

$$\frac{d}{dx} f(x)g(x) = f'(x)g(x) + f(x)g'(x)$$

Find the *derivative* of the following function

Example 1.9

$$f(x) = \frac{x^5}{2x + 7}$$

Quotient Rule

$$\frac{d}{dx}\frac{f(x)}{g(x)} = \frac{g(x)f'(x) - f(x)g'(x)}{\big(g(x)\big)^2}$$

Find the *derivative* of the following function

Example 1.10

$f(x) = e^{-2x+7}$

Chain Rule

$$\frac{d}{dx} f\big(g(x)\big) = f'\big(g(x)\big)g'(x)$$

Find the *derivative* of the following function

Example 1.11

$f(x) = \ln(x^2)$

Integration

Find the *antiderivative* of the following function

Example 2.1

$f(x) = 2$

Find the *antiderivative* of the following function

Example 2.2

$f(x) = x$

Integration of x to a constant

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c$$

Find the *antiderivative* of the following function

Example 2.3

$f(x) = \frac{1}{x}$

Find the *antiderivative* of the following function

Example 2.4

$f(x) = x^{3/2}$

Find the *antiderivative* of the following function

Example 2.5

$f(x) = e^x$

Find the *antiderivative* of the following function

Example 2.6

$f(x) = e^{-x}$

Find the *antiderivative* of the following function

Example 2.7

$f(x) = e^{-2x}$

Solve the following integral

Example 3.1

$$\int_0^1 (2x + x^5)dx$$

Solve the following integral

Example 3.2

$$\int_2^3 e^{-x} dx$$

U-substitution

$$\int f\big(g(x)\big)g'(x)dx = \int f(u)dx$$

Solve the following integral

Example 3.3

$$\int_2^3 xe^{x^2}\,dx$$

Solve the following integral

Example 3.4

$$\int_0^\infty xe^{-x}dx$$

Integrating by Parts

$$\int f(x)g'(x)dx = f(x)g(x) - \int f'(x)g(x)dx$$

OR

$$\int_a^b udv = uv\Big|_a^b - \int_a^b vdu$$

Solve the following integral

Example 3.5

$$\int_1^2 x^2 \ln(x) dx$$

Solve the following integral

Example 3.6

$$\int_1^2 \ln(x)dx$$

Solve the following integral

Example 3.7

$$\int_1^2 x^2 e^x \, dx$$

Pre Chapter 24 Slides