Chapter 3: Independent Events

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2024-10-07

Learning objectives

1. Define independence of 2-3 events given probability notation
2. Calculate whether two or more events are independent

Where are we?

Basics of probability

- Outcomes and events
- Sample space
- Probability axioms
- Probability properties
- Counting
- Independence
- Conditional probability
- Bayes' Theorem
- Random Variables

Probability for discrete random variables

- Functions: pmfs/CDFs
- Important distributions
- Joint distributions
- Expected values and variance

Probability for continuous random variables

- Calculus
- Functions: pdfs/CDFs
- Important distributions
- Joint distributions
- Expected values and variance

Advanced probability

- Central limit theorem
- Functions: moment generating functions

Independent Events

Definition: Independence

Events A and B are **independent** if

$$\mathbb{P}(A \cap B) = \mathbb{P}(A) \cdot \mathbb{P}(B).$$

Notation: For shorthand, we sometimes write $A \perp\!\!\!\perp B$, to denote that A and B are independent events.

- Also note:

$$\begin{aligned} \mathbb{P}(A \cap B) = \mathbb{P}(A) \cdot \mathbb{P}(B) &\implies A \perp\!\!\!\perp B \\ A \perp\!\!\!\perp B &\implies \mathbb{P}(A \cap B) = \mathbb{P}(A) \cdot \mathbb{P}(B) \end{aligned}$$

Example of two dice

Example 1

Two dice (red and blue) are rolled. Let A = event a total of 7 appears, and B = event red die is a six. Are events A and B independent?

Independence of 3 Events

Definition: Independence of 3 Events

Events A, B, and C are *mutually* **independent** if

1. - $\mathbb{P}(A \cap B) = \mathbb{P}(A) \cdot \mathbb{P}(B)$
   - $\mathbb{P}(A \cap C) = \mathbb{P}(A) \cdot \mathbb{P}(C)$
   - $\mathbb{P}(B \cap C) = \mathbb{P}(B) \cdot \mathbb{P}(C)$
2. $\mathbb{P}(A \cap B \cap C) = \mathbb{P}(A) \cdot \mathbb{P}(B) \cdot \mathbb{P}(C)$

Remark:

On your homework you will show that $(1) \not\Rightarrow (2)$ and $(2) \not\Rightarrow (1)$.

Probability at least one smoker

Example 2

Suppose you take a random sample of n people, of which people are smokers and non-smokers independently of each other. Let

- A_i = event person i is a smoker, for $i = 1, \ldots, n$, and
- p_i = probability person i is a smoker, for $i = 1, \ldots, n$.

Find the probability that at least one person in the random sample is a smoker.