Chapter 7: Discrete vs. Continuous Random Variables

Meike Niederhausen and Nicky Wakim

2024-10-14

Learning Objectives

1. Map the sample space to the set of real numbers using a discrete and continuous random variable
2. Distinguish between discrete and continuous random variables from a written description

Where are we?

Basics of probability

- Outcomes and events
- Sample space
- Probability axioms
- Probability properties
- Counting
- Independence
- Conditional probability
- Bayes' Theorem
- Random Variables

Probability for discrete random variables

- Functions: pmfs/CDFs
- Important distributions
- Joint distributions
- Expected values and variance

Probability for continuous random variables

- Calculus
- Functions: pdfs/CDFs
- Important distributions
- Joint distributions
- Expected values and variance

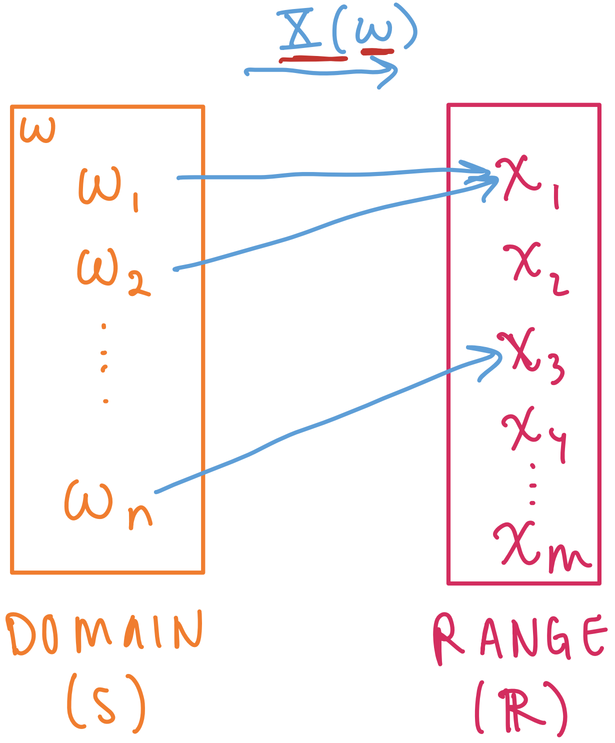
Advanced probability

- Central limit theorem
- Functions: moment generating functions

What is a random variable?

Definition: Random Variable

For a given sample space S, a **random variable** (r.v.) is a function whose domain is S and whose range is the set of real numbers $\mathbb{R}$. A random variable assigns a real number to each outcome in the sample space.



$X \rightarrow$ capital X

$$\underbrace{X(\omega)}_{\text{RV function}} = x$$

outcome of RV

Let's demonstrate this definition with our coin toss

Example 1 order matters

Suppose we toss 3 fair coins. H or T

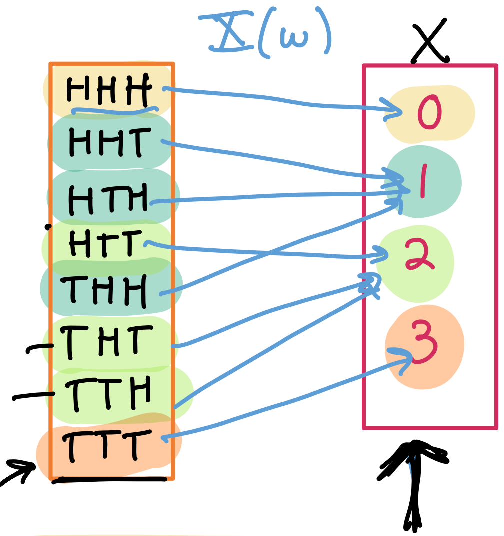
1. What is the sample space?
2. What are the probabilities for each of the elements in the sample space?
3. What are the probabilities that you get 0, 1, 2, or 3 tails?

① $S = \{HHH, HHT, HTT, HTH, THH, THT, TTH, TTT\}$

② $P(HHH) = \frac{|HHH|}{|S|} = \frac{1}{8}$

same prob for all others

③ let X = # of tails



multiple arrows to diff range values

BUT one outcome in domain cannot be mapped to multiple range values

$P(X=0) = P(HHH) = \frac{1}{8}$

$P(X=1) = P(HHT) + P(HTH) + P(THH) = \frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \frac{3}{8}$

$P(X=2) = 3\left(\frac{1}{8}\right) = \frac{3}{8}$

$P(X=3) = \frac{1}{8}$

Let's stretch our definition of random variables

Example 2

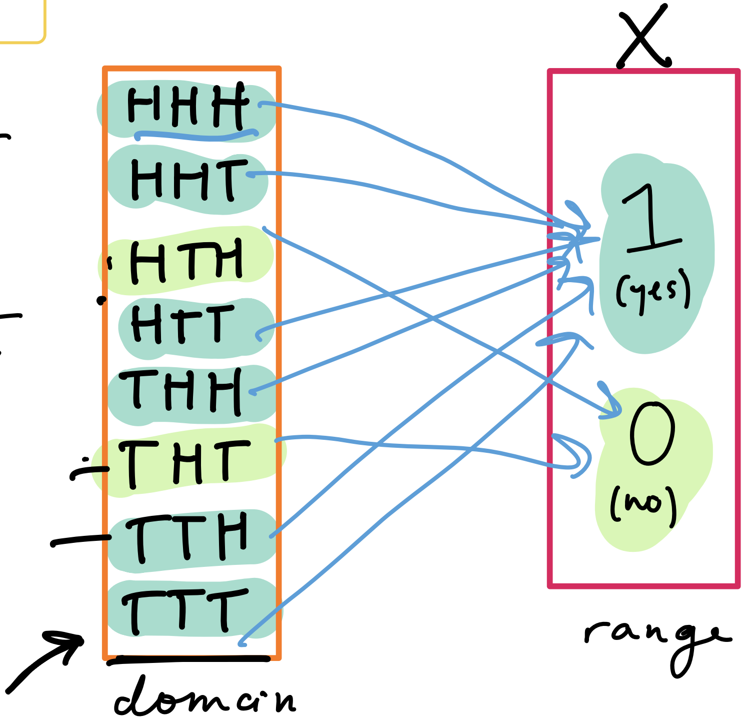
What are some other random variables we could consider in Example 1?

$Y(\omega) = y$ that we get two in a row of the same heads/tails

yes or no (1 or 0)

$P(Y=0) = \frac{2}{8} = \frac{1}{4}$

$P(Y=1) = \frac{6}{8} = \frac{3}{4}$



Some remarks on random variables

- A random variable's value is completely determined by the outcome ω, where $\omega \in S$
  - What is *random* is the outcome ω
- A random variable is a function from the sample space (with outcomes ω) to the set of real numbers
  - We typically write X instead of $X(\omega)$, where X is our random variable
- For example, if we roll three dice, there are $6^3 = 216$ possible outcomes (which is ω)
  - We can define a random variable as the sum of the of the three dice
  - If our outcome is the set of numbers the dice landed on ($\omega = (a, b, c)$), then

$$X(\omega) = X = a + b + c$$

Let's look at a continuous R.V.

Example 3

Let X = how many hours you slept last night. (including decimals)

1. What is the sample space S?
2. What is the range of possible values for X?
3. What is $X(\omega)$? .

① $S = \{\omega \geq 0\}$

② ③ $X(\omega) = 1\omega$

$X(\omega) \geq 0$

$X \geq 0$

$X \in [0, \infty)$

$X(\omega) = \omega$

outcome in Range

outcome from SS

$X(w) = \underline{a} + \underline{b} + \underline{c} + \underline{d} + e + f + g$

6   7 or 3

Discrete vs. Continuous r.v.'s

- For a **discrete** r.v., the set of possible values is either finite or can be put into a countably infinite list
  - You could *theoretically* list the specific possible outcomes that the variable can take
  - If you sum the rolls of three dice, you must get a whole number. For example, you can't get any number between 3 and 4.

Σ

- **Continuous** r.v.'s take on values from continuous *intervals*, or unions of continuous intervals
  - Variable takes on a range of values, but there are infinitely possible values within the range
  - If you keep track of the time you sleep, you can sleep for 8 hours or 7.9 hours or 7.99 hours or 7.999 hours …