Pre Chapter 24: Calculus Review

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Learning Objectives

1. Find derivatives of continuous functions with one variable
2. Find antiderivatives and integrals of functions with one variable

Where are we?

Basics of probability

- Outcomes and events
- Sample space
- Probability axioms
- Probability properties
- Counting
- Independence
- Conditional probability
- Bayes' Theorem
- Random Variables

Probability for discrete random variables

- Functions: pmfs/CDFs
- Important distributions
- Joint distributions
- Expected values and variance

Probability for continuous random variables

- Calculus
- Functions: pdfs/CDFs
- Important distributions
- Joint distributions
- Expected values and variance

Advanced probability

- Central limit theorem
- Functions: moment generating functions

Differentiation

Find the *derivative* of the following function

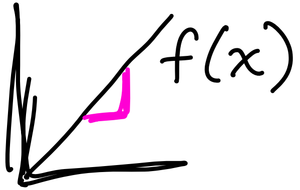
slope of fn $\frac{d}{dx}(f(x))$ $f'(x)$

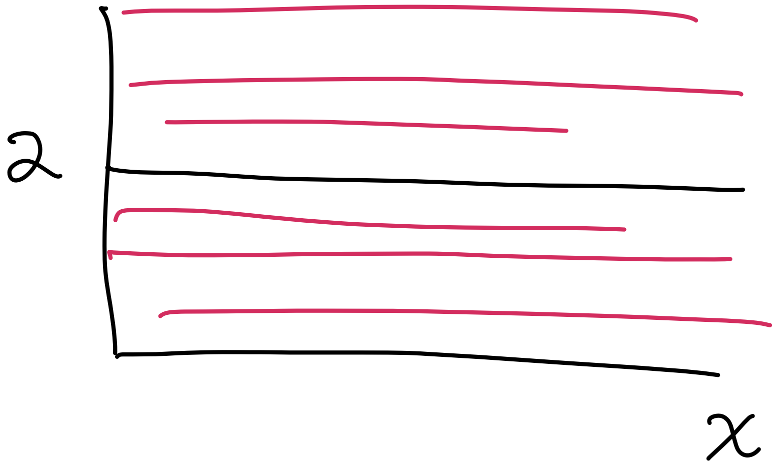
Example 1.1

$f(x) = 2$

Derivative of a constant

$$\frac{d}{dx}c = 0$$





slope is flat

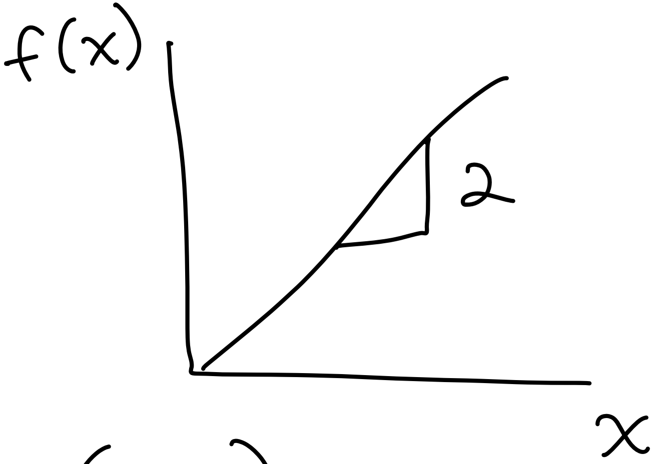
$$\frac{d}{dx}(2) = 0$$

$$f'(x) = \frac{d}{dx}f(x) = \frac{d}{dx}(2) = 0$$

Find the *derivative* of the following function

Example 1.2

$f(x) = 2x$



$$f'(x) = 2$$

$$\frac{d}{dx}\left(f(x)\right) = \frac{d}{dx}(2x) = 2$$

Find the *derivative* of the following function

Example 1.3

$f(x) = 2x + 2$

$$f'(x) = \frac{d}{dx}(2x + 2)$$

$$= \frac{d}{dx}(2x) + \frac{d}{dx}(2)$$

$$= 2 + 0$$

$$f'(x) = 2$$

Find the *derivative* of the following function

Example 1.4

$f(x) = x^2$

Derivative of x to a constant

$$\frac{d}{dx}x^n = nx^{n-1}$$

$$f'(x) = \frac{d}{dx}(x^2) = 2x^{\overbrace{2-1}^{=1}}$$

$$= 2x$$

Find the *derivative* of the following function

Example 1.5

$f(x) = 3\sqrt{x} + \frac{2}{x} + 5$

$$f'(x) = \frac{d}{dx}\left(3x^{1/2} + 2x^{-1} + 5\right)$$

$$= \frac{d}{dx}\left(3x^{1/2}\right) + \frac{d}{dx}\left(2x^{-1}\right) + \frac{d}{dx}(5)$$

$$= \frac{1}{2}\cdot 3x^{1/2-1} + (-1)(2)x^{-1-1} + 0$$

$$= \frac{3}{2}x^{-1/2} - 2x^{-2}$$

$$f'(x) = \frac{3}{2\sqrt{x}} - \frac{2}{x^2}$$

Find the *derivative* of the following function

Example 1.6

$f(x) = e^x$

Derivative of exponential function

$$\frac{d}{dx}e^x = e^x$$

$$f'(x) = \frac{d}{dx}(e^x) = e^x$$

Find the *derivative* of the following function

Example 1.7

$f(x) = \ln(x)$

Derivative of logarithm

$$\frac{d}{dx} ln(x) = \frac{1}{x}$$

$f'(x) = \frac{1}{x}$

★ ln or log $\xrightarrow{\text{assume both}}$ natural log

$\log_e$

$\cancel{\log_{10}}$

Find the *derivative* of the following function

Example 1.8

$f(x) = x^2 e^x$ $\quad f(x) = g(x)h(x)$

Product Rule

$$\frac{d}{dx} f(x)g(x) = f'(x)g(x) + f(x)g'(x)$$

$$f(x) = \underbrace{x^2}_{g(x)} \underbrace{e^x}_{h(x)}$$

$$f'(x) = g'(x)h(x) + g(x)h'(x)$$

$$= (2x)(e^x) + x^2(e^x)$$

$$f'(x) = 2xe^x + x^2e^x$$

Find the *derivative* of the following function

$f'(x) = 5x^{5-1} = 5x^4$

Example 1.9

$$h(x) = \frac{x^5}{2x+7} = \frac{f(x)}{g(x)} \rightarrow$$

Quotient Rule

$$\frac{d}{dx}\frac{f(x)}{g(x)} = \frac{g(x)f'(x) - f(x)g'(x)}{\left(g(x)\right)^2}$$

$$\frac{\text{lo d hi} - \text{hi d lo}}{\text{lolo}}$$

$$h'(x) = \frac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2}$$

$$= \frac{(2x+7)(5x^4) - (x^5)(2)}{(2x+7)^2}$$

$$= \frac{10x^5 + 35x^4 - 2x^5}{(2x+7)^2}$$

$$\boxed{h'(x) = \frac{8x^5 + 35x^4}{(2x+7)^2}}$$

Find the *derivative* of the following function

Example 1.10

$f(x) = e^{-2x+7}$ $\qquad e^{g(x)}$

Chain Rule

$$\frac{d}{dx} f\left(g(x)\right) = f'\left(g(x)\right) g'(x)$$

$$f(x) = e^{g(x)}$$

$$\downarrow d/dx$$

$$f'(x) = e^{g(x)} \, g'(x)$$

$$f'(x) = e^{-2x+7}\left(\frac{d}{dx}(-2x+7)\right)$$

$$= e^{-2x+7}(-2)$$

$$f'(x) = -2e^{-2x+7}$$

Find the *derivative* of the following function

Example 1.11

$f(x) = \ln(x^2)$

$g(x) = x^2$

$$f'(x) = f'(g(x))\, g'(x)$$

$$= \frac{1}{x^2} \cdot \left(\frac{d}{dx} x^2\right)$$

$$= \frac{1}{\cancel{x^2}} \cdot 2\cancel{x}$$

$$f'(x) = \frac{2}{x}$$

Integration

Find the *antiderivative* of the following function

Example 2.1

$f'(x) = 2$

deriving

$$\int_0^0 2\,dx = 2x + C$$

integrating

anti-deriv.

$f'(x) = 2$ what's $f(x)$?

$$\boxed{f(x) = \frac{2x}{2} + C}$$

Find the *antiderivative* of the following function

Example 2.2

$f'(x) = x$

Integration of x to a constant

$$\int \overset{f'(x)}{x^n}\, dx = \frac{x^{n+1}}{n+1} + c$$

$$f(x) = \int 1x^1\, dx + c$$

$$= \frac{1}{2} x^{1+1}$$

$$\boxed{f(x) = \frac{1}{2} x^2 + c}$$

check

$$f'(x) = \frac{d}{dx}\left(\frac{1}{2} x^2 + c\right)$$

$$= \frac{2 \cdot \frac{1}{2}}{} x^{2-1} + 0$$

$$= 1x^1 = x$$

$$1x^1$$

$$\frac{d}{dx}(x^n) = n x^{n-1}$$

$$\frac{2}{2} = 1$$

Find the *antiderivative* of the following function

Example 2.3

$f(x) = \frac{1}{x}$

$$\frac{d}{dx}(\log x) = \frac{1}{x}$$

$\ln x$

$$\int_0^0 \frac{1}{x}\,dx = \ln|x| + c$$

$$\frac{d}{dx}\underbrace{\ln(-x)}_{g(x)} = \frac{1}{(-x)}(-1) = \frac{1}{x}$$

Find the *antiderivative* of the following function

Example 2.4

$f(x) = x^{3/2}$

$\rightarrow \quad \int x^{n}\, dx = \dfrac{x^{n+1}}{n+1} + C$

Find the *antiderivative* of the following function

Example 2.5

$f(x) = e^x$

$\frac{d}{dx} e^x = e^x$

$\int e^x \, dx = \boxed{e^x + C}$

Find the *antiderivative* of the following function

Example 2.6

$f(x) = e^{-x}$

$g(x)$

$$\int e^{-x}\,dx = -1e^{-x} + c$$

$$\frac{d}{dx} e^{-x} = e^{-x}(-1)$$

$$= -e^{-x}$$

$$\frac{d}{dx}(-1)e^{-x} = (-1e^{-x})(-1)$$

$$= e^{-x}$$

$$= \boxed{-e^{-x} + c}$$

Find the *antiderivative* of the following function

Example 2.7

$f(x) = e^{-2x}$

1

$g(x) = -2x$

$g'(x) = -2$

$$\int e^{-2x}\,dx = \boxed{\frac{-1}{2}} e^{-2x} + C$$

$$\boxed{} \times g'(x) = 1$$

$$-\frac{1}{2} \times -2 = 1$$

$$= \boxed{-\frac{1}{2} e^{-2x} + C}$$

Solve the following integral

Example 3.1

$$\int_0^1 (2x + x^5)dx$$

$$\frac{d}{dx} x^2 = 2x$$

$$\int_0^1 (2x + x^5)\,dx$$

$$= x^2 + \frac{1}{6}x^6 \Big|_{x=0}^{x=1}$$

$$= \left[(1)^2 + \frac{1}{6}(1)^6\right]_{x=1} - \cancel{\left[0^2 + \frac{1}{6}0^6\right]_{x=0}}$$

0

$$= 1 + \frac{1}{6}(1) - 0$$

$$= 1 + \frac{1}{6} = \boxed{\frac{7}{6}}$$

Solve the following integral

$u = -x$

$$\frac{du}{dx} = \frac{d}{dx}(u) = \frac{d}{dx}(-x) = -1$$

$$\frac{du}{dx} = -1$$

$$du = -1\,dx$$

$$dx = -1\,du$$

Example 3.2

$$\int_2^3 e^{-x}\,dx$$

$g(x)$

$$-e^{-x}\Big|_{x=2}^{x=3} = -e^{-3} - (-e^{-2})$$

U-substitution

$$\int \underbrace{f(g(x))}_{e^{-x}}\underbrace{g'(x)}_{g'(x)}dx = \int f(u)\,du$$

$e^{-x} = f(g(x))\,g'(x)$

find the anti deriv

$$\int_2^3 e^{-x}\,dx = \int_{-2}^{-3} e^{u}(-1)\,du$$

$u = -x$

$u = -3$ top

$u = -2$ bottom

$$= \int_{-2}^{-3} -e^{u}\,du = -\int_{-2}^{-3} e^{u}\,du = -1\left[e^{u}\Big|_{u=-2}^{u=-3}\right] = -1\left(e^{-3} - e^{-2}\right)$$

$$= \boxed{e^{-2} - e^{-3}}$$

Solve the following integral

Example 3.3 u sub

$$\int_2^3 xe^{x^2}\,dx$$

$$u = x^2$$

$$\frac{1}{2}\frac{du}{dx} = 2x$$

$$x\,dx = \frac{1}{2}\,du$$

$$\int_2^3 e^{x^2}x\,dx =$$

$$\int e^{u}\frac{1}{2}du$$

bounds?

$x = 2: \quad u = (2)^2 = 4$

$x = 3: \quad u = (3)^2 = 9$

$$\int_{u=4}^{u=9}\frac{1}{2}e^{u}\,du = \frac{1}{2}e^{u}\Big|_{u=4}^{u=9}$$

$$= \frac{1}{2}e^{9} - \frac{1}{2}e^{4} = \boxed{\frac{1}{2}\left(e^{9}-e^{4}\right)}$$

Solve the following integral

Example 3.4

$$\int_0^\infty xe^{-x}\,dx$$

good to have der that goes to constant

Integrating by Parts

$$\int f(x)g'(x)dx = f(x)g(x) - \int f'(x)g(x)dx$$

OR

$$\int_a^b u\,dv = uv\Big|_a^b - \int_a^b v\,du$$

$u = x$

$\frac{du}{dx} = 1$

$du = 1\,dx$

$dv = e^{-x}\,dx$

$v = \int e^{-x}dx = -e^{-x}$

$$\int_0^\infty x e^{-x}\,dx = \left[x(-e^{-x})\Big|_0^\infty \right] - \int_0^\infty -e^{-x}\,1\,dx$$

$$= \left[\underbrace{(\infty)}_{\infty}\underbrace{(-e^{-\infty})}_{0} - \underbrace{(0)}_{0}(-\underbrace{e^{-0}}_{1}) \right] - \int_0^\infty -e^{-x}dx$$

$$= [0 - 0] + \left[-e^{-x}\right]_0^\infty$$

$$= -e^{-\infty} - (-e^{-0}) = 0 + 1 = \boxed{1}$$

Solve the following integral

Example 3.5

$$\int_1^2 x^2 \ln(x) dx$$

Solve the following integral

Example 3.6

$$\int_1^2 \ln(x)dx$$

Solve the following integral

from last year's slides:

Solve the following integral

Example 3.7

$$\int_1^2 x^2 e^x \, dx$$

e^x

$f(x) = x^2$

$f'(x) = 2x$

$f''(x) = 2$

$u = x^2$ $\quad$ $dv = e^x \, dx$

$\frac{du}{dx} = 2x$ $\quad$ $v = \int e^x \, dx = e^x$

$du = 2x \, dx$

$$\int_1^2 x^2 e^x \, dx = \underline{x^2 e^x \Big|_1^2} - \underline{\int_1^2 2x e^x \, dx}$$

$$= 2^2 e^2 - 1^2 e^1 - 2e^2$$

$$= 4e^2 - e^1 - 2e^2$$

$$= \boxed{2e^2 - e}$$

I by P #2:

$u = 2x$ $\quad$ $dv = e^x dx$

$\frac{du}{dx} = 2$ $\quad$ $v = e^x$

$$= 2x e^x \Big|_1^2 - \int_1^2 2e^x dx$$

$$= 4e^2 - 2e^1 - \underline{2e^x \Big|_1^2}$$

$$= 2e^2$$