

Chapter 26: Independent Continuous RVs

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Learning Objectives

1. Show that a joint pdf consists of two independent, continuous RVs.
2. Combine two independent RVs into one joint pdf or CDF.

How do we represent independent continuous RVs in a joint pdf?

What do we know about independence for events and discrete RVs?

For events: If $A \perp B$

$$P(A \cap B) = P(A)P(B)$$

$$P(A|B) = P(A)$$

For discrete RVs: If $X \perp Y$

$$p_{X,Y}(x, y) = p_X(x)p_Y(y)$$

$$F_{X,Y}(x, y) = F_X(x)F_Y(y)$$

$$p_{X|Y}(x|y) = p_X(x)$$

$$p_{Y|X}(y|x) = p_Y(y)$$

What does it mean for continuous r.v.'s to be independent?

For continuous RVs: If $X \perp Y$

Constructing a joint pdf from two independent, continuous RVs

Example 1.1

Let X and Y be independent r.v.'s with $f_X(x) = \frac{1}{2}$, for $0 \leq x \leq 2$ and $f_Y(y) = 3y^2$, for $0 \leq y \leq 1$.

1. Find $f_{X,Y}(x, y)$.

Constructing a joint pdf from two independent, continuous RVs

Example 1.2

Let X and Y be independent r.v.'s with $f_X(x) = \frac{1}{2}$, for $0 \leq x \leq 2$ and $f_Y(y) = 3y^2$, for $0 \leq y \leq 1$.

2. Find

$$\mathbb{P}(0 \leq X \leq 1, 0 \leq Y \leq \frac{1}{2})$$

.

Showing independence from joint pdf

Example 2.1

Let $f_{X,Y}(x, y) = 18x^2y^5$, for
 $0 \leq x \leq 1, 0 \leq y \leq 1$.

1. Are X and Y independent?

Finding CDF from two independent RVs

Example 2.2

Let $f_{X,Y}(x, y) = 18x^2y^5$, for
 $0 \leq x \leq 1, 0 \leq y \leq 1$.

2. Find $F_{X,Y}(x, y)$.

Showing independence from joint pdf

Do this problem at home for extra practice. The solution is available in Meike's video!

Example 3

Let $f_{X,Y}(x, y) = 2e^{-(x+y)}$, for $0 \leq x \leq y$. Are X and Y independent?

Final statement on independence

1. If $f_{X,Y}(x, y) = g(x)h(y)$, where $g(x)$ and $h(y)$ are pdf's, then X and Y are independent.
 - The domain of the joint pdf needs to be independent as well!!
2. If $F_{X,Y}(x, y) = G(x)H(y)$, where $G(x)$ and $H(y)$ are cdf's, then X and Y are independent.
 - The domain of the joint CDF needs to be independent as well!!

