Chapter 28: Expected Values of Continuous Random Variables

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Learning Objectives

1. Calculate the mean (expected value) of a continuous RV

Expected value of a function of a continuous RV

How do we calculate expected values of discrete RVs?

For discrete RVs: weight average

$$\mathbb{E}[X] = \sum_{i=1}^{n} x_i p_X(x_i).$$

How do we calculate expected values of continuous RVs?

For continuous RVs:

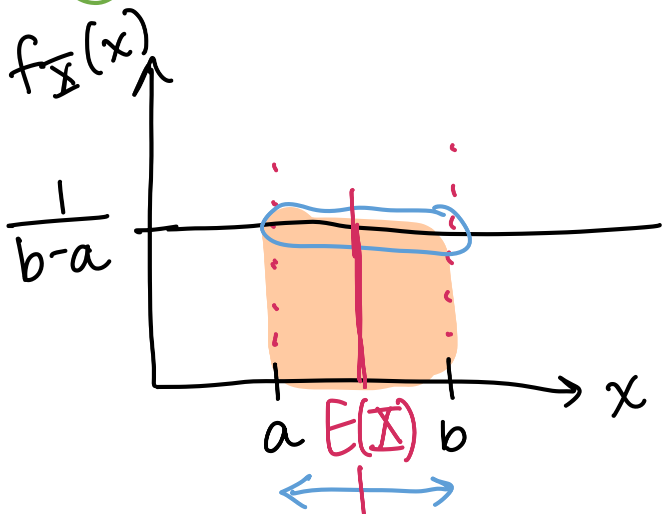
$$E(X) = \int_{-\infty}^{\infty} x \, f_X(x) \, dx$$

adjust integrands based of bounds of $f_X(x)$ (pdf)

Expected Value of the Uniform Distribution (cont form)

Example 1

Let $f_X(x) = \frac{1}{b-a}$, for $a \leq x \leq b$. Find $\mathbb{E}[X]$.



$$E(X) = \frac{a+b}{2}$$

$$E(X) = \int_a^b x \left(\frac{1}{b-a}\right) dx$$

$$= \underbrace{\left(\frac{1}{b-a}\right)\frac{1}{2}}\, x^2 \Big|_{x=a}^{x=b}$$

$$= \frac{2}{b-a}\left[b^2 - a^2\right]$$

$$= \frac{2}{\cancel{b-a}}(b+a)\cancel{(b-a)}$$

$$= \frac{b+a}{2}$$

Expected Value of the Exponential Distribution

$X \sim \mathrm{Exp}(\lambda)$

Example 2

Let $f_X(x) = \lambda e^{-\lambda x}$, for $x > 0$ and $\lambda > 0$. Find $\mathbb{E}[X]$.

Integrating by Parts

$$\int_a^b u\,dv = uv\Big|_a^b - \int_a^b v\,du$$

int by parts:

$u = \lambda x \quad dv = 1e^{-\lambda x}dx$

$du = \lambda\, dx \qquad -\frac{1}{\lambda}e^{-\lambda x}$

$\frac{d}{dx}(u) = \frac{d}{dx}(\lambda x)$

$\frac{du}{dx} = \lambda$

$$E(X) = \int_{-\infty}^{\infty} x f_X(x)\,dx$$

$$= \int_0^\infty x \lambda e^{-\lambda x} dx$$

$$= \cancel{\lambda} x\left[-\frac{1}{\cancel{\lambda}}e^{-\lambda x}\right]\Big|_0^\infty - \int_0^\infty \left(-\frac{1}{\cancel{\lambda}}e^{-\lambda x}\right)\cancel{\lambda}\,dx$$

$x \to \infty$ slower than $e^{-\lambda x} \to 0$

$$= 0 - 0 + \int_0^\infty e^{-\lambda x}dx$$

$$= -\frac{1}{\lambda}e^{-\lambda x}\Big|_{x=0}^{x=\infty} = -\frac{1}{\lambda}(0) - \left(-\frac{1}{\lambda}\right)e^{-0}$$

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$$= \boxed{\frac{1}{\lambda}}$$

calc prob: $P(a<X\leq b)$

$\int_a^b f_X(x)\,dx$

calc exp val:

$\int_{-\infty}^{\infty} x f_X(x)\,dx$