# Lesson 3: Defining Probability

TB sections 2.1

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# Learning Objectives

1. Define probability and explain the Law of Large Numbers within examples
2. Define relationships between events and their probability properties (including disjoint events, non-disjoint events, complements, and independent events)
3. Calculate an unknown probability in a word problem using the probability properties

# Where are we?

## Data
- Collecting data
- Categorical vs. Numeric
- Summary statistics
- Data visualization

## Probability
- Probability rules
- Independence, conditional
- Random variables and probability distributions
- Linear combinations
- Binomial, Normal, and Poisson

## Sampling Variability, and Statistical Inference
- Sampling distributions
- Central Limit Theorem
- Confidence Intervals
- Hypothesis tests

## Inference for continuous data/outcomes
- One sample t-test
- 3+ independent samples
- Simple linear regression / correlation
- 2 sample tests: paired and independent
- Power and sample size
- Non-parametric tests

## Inference for categorical data/outcomes
- One proportion test
- Fisher's exact test
- Non-parametric tests
- Chi-squared test
- 2 proportion test
- Power and sample size

## R
- Basics
- Reproducibility
- Quarto
- Packages
- Data visualization
- Data wrangling
- R Projects
- ...

# Let’s start with an example!

**Example: Rolling fair 6-sided dice**

Suppose you roll a fair 6-sided die.

1. What is the probability that you roll a 4?
2. What is the probability that you roll an even number?
3. What is the probability that you did not roll a 3?

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# What is a probability?

**Definition: Probability**

How likely something will happen.

- On a more technical note, the probability of an outcome is the proportion of times the outcome would occur if the random phenomenon could be observed an infinite number of times.

- We can think of flipping a coin. There are two possible outcomes (heads or tails). The probability of getting heads is 0.5.

# What is a probability? with the Law of Large Numbers

- We can think of flipping a coin. There are two possible outcomes (heads or tails). The probability of getting heads is 0.5.
  - If we flip the coin 10 times, it is not certain that we will get 5 heads. However, if we flip it infinite times, we will get heads 50% of the flips.

- Fun "Seeing Theory" demonstration!

**Law of large numbers**

As more observations are collected, the proportion of occurrences, $\hat{p}$, with a particular outcome converges to the true probability $p$ of that outcome.

# Poll Everywhere Question 1

# Some probability notation

- Probability typically defined as a proportion
  - Takes values between 0 and 1

- Probability can also be expressed as a "percent chance," taking values between 0% and 100%

- If we want to discuss the probability of an event, say A, we would write $P(A)$
  - We can write: $A = \{\text{rolling a 1}\}$, with associated probability $P(A)$
  - OR we can write $P(\text{rolling a 1})$

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# Disjoint / mutually exclusive events

**Disjoint / mutually exclusive events**

Two events or outcomes are called **disjoint** or **mutually exclusive** if they cannot both happen at the same time.

# Poll Everywhere Question 2

# Probability for disjoint events

**Probability rule for disjoint events**

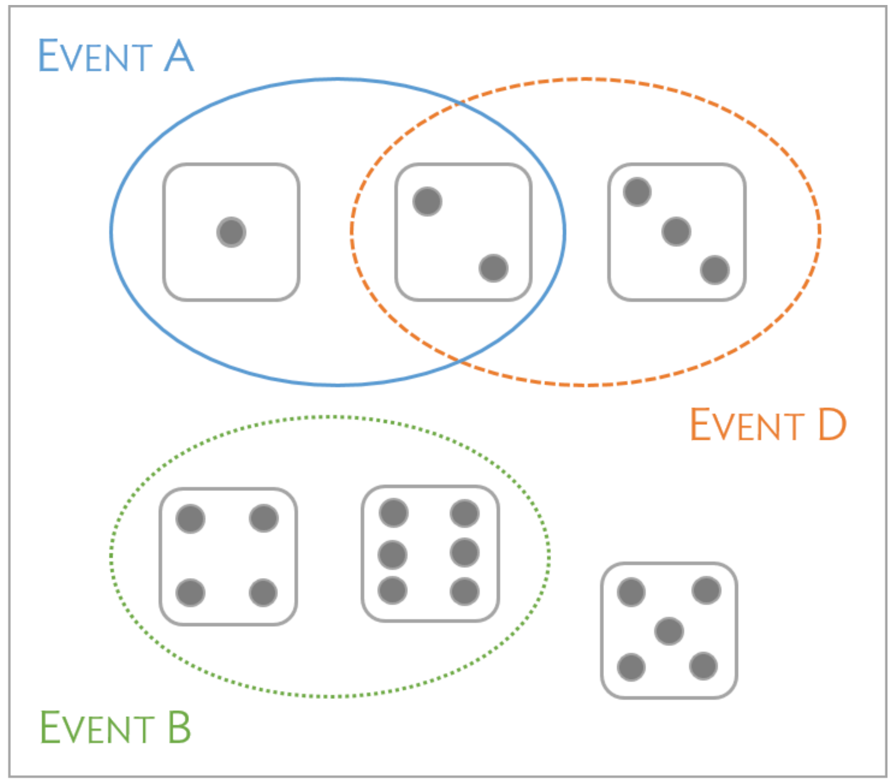
If $A_1$ and $A_2$ represent two disjoint outcomes, then the probability that either one of them occurs is given by

$$P(A_1 \text{ or } A_2) = P(A_1) + P(A_2)$$

If there are $k$ disjoint outcomes $A_1$, ..., $A_k$, then the probability that either one of these outcomes will occur is

$$P(A_1) + P(A_2) + \cdots + P(A_k)$$

- From the poll everywhere question with the die, what is the probability of event A or B?



# Probabilities when events are not disjoint

- When events are not disjoint, we cannot use the previous addition rule for probabilities!!
- We must use a general rule that recognizes the potential overlap between events

**General probability addition rule**

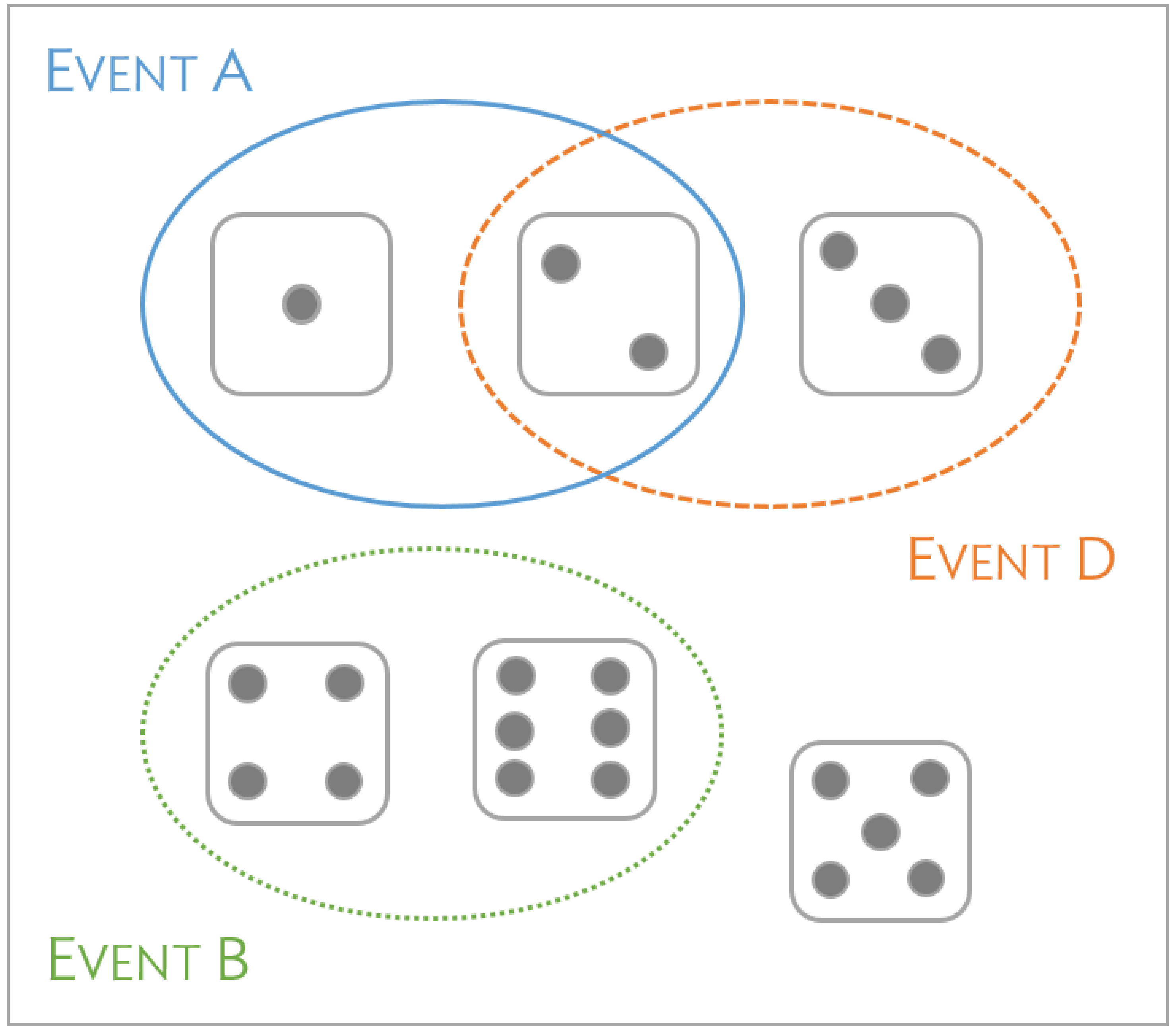
If $A$ and $B$ are any two events, disjoint or not, then the probability that at least one of them will occur is

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B),$$

where $P(A \text{ and } B)$ is the probability that both events occur.

# Think back to our die

- Event A and D are not disjoint, they share an outcome of rolling a 2
- How do we find the probability of event A or event D?



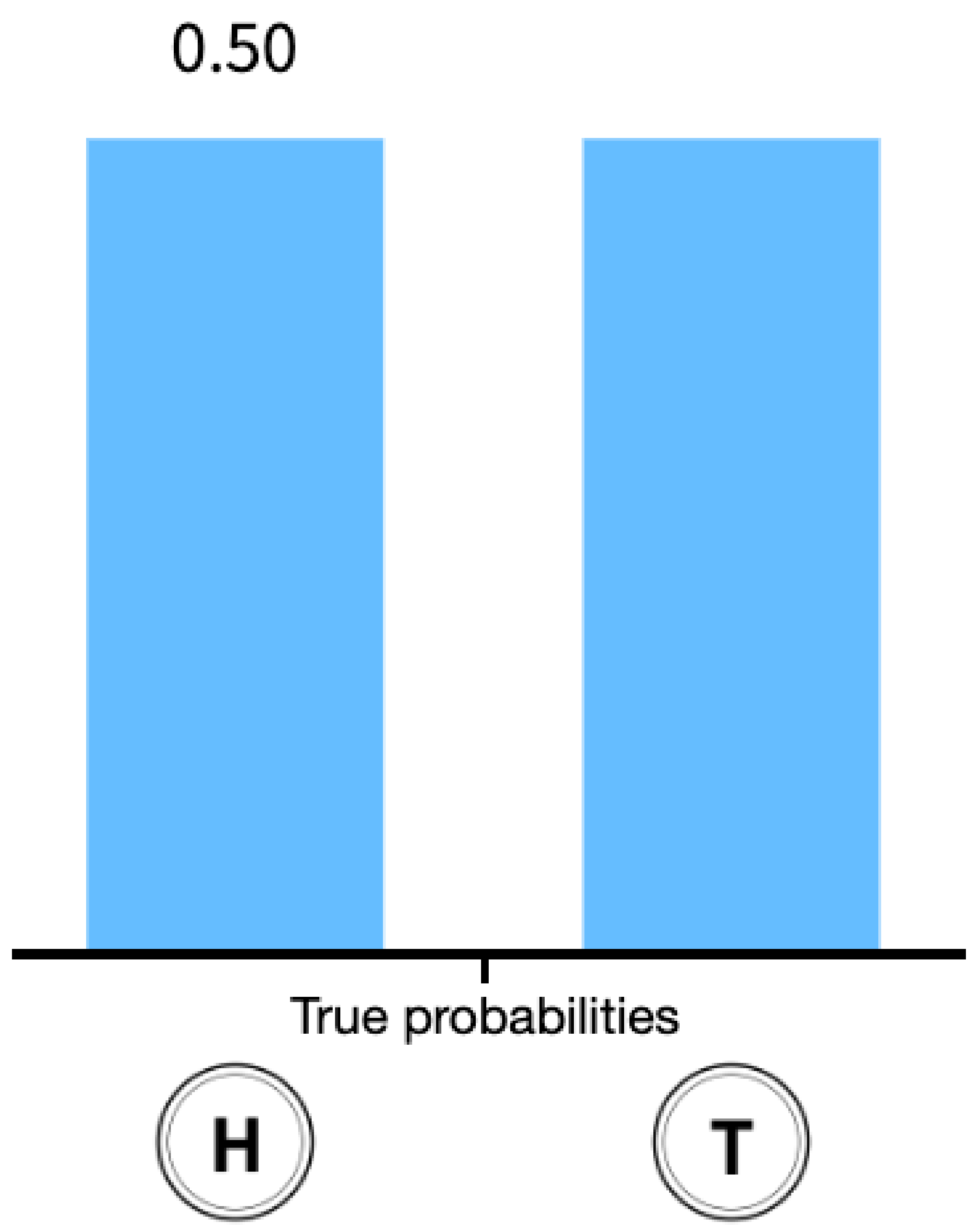
# Probability distributions

- A **probability distribution** consists of all disjoint outcomes and their associated probabilities
- We've already seen one in our heads and tails example

**Rules for a probability distribution**

A probability distribution is a list of all possible outcomes and their associated probabilities that satisfies three rules:

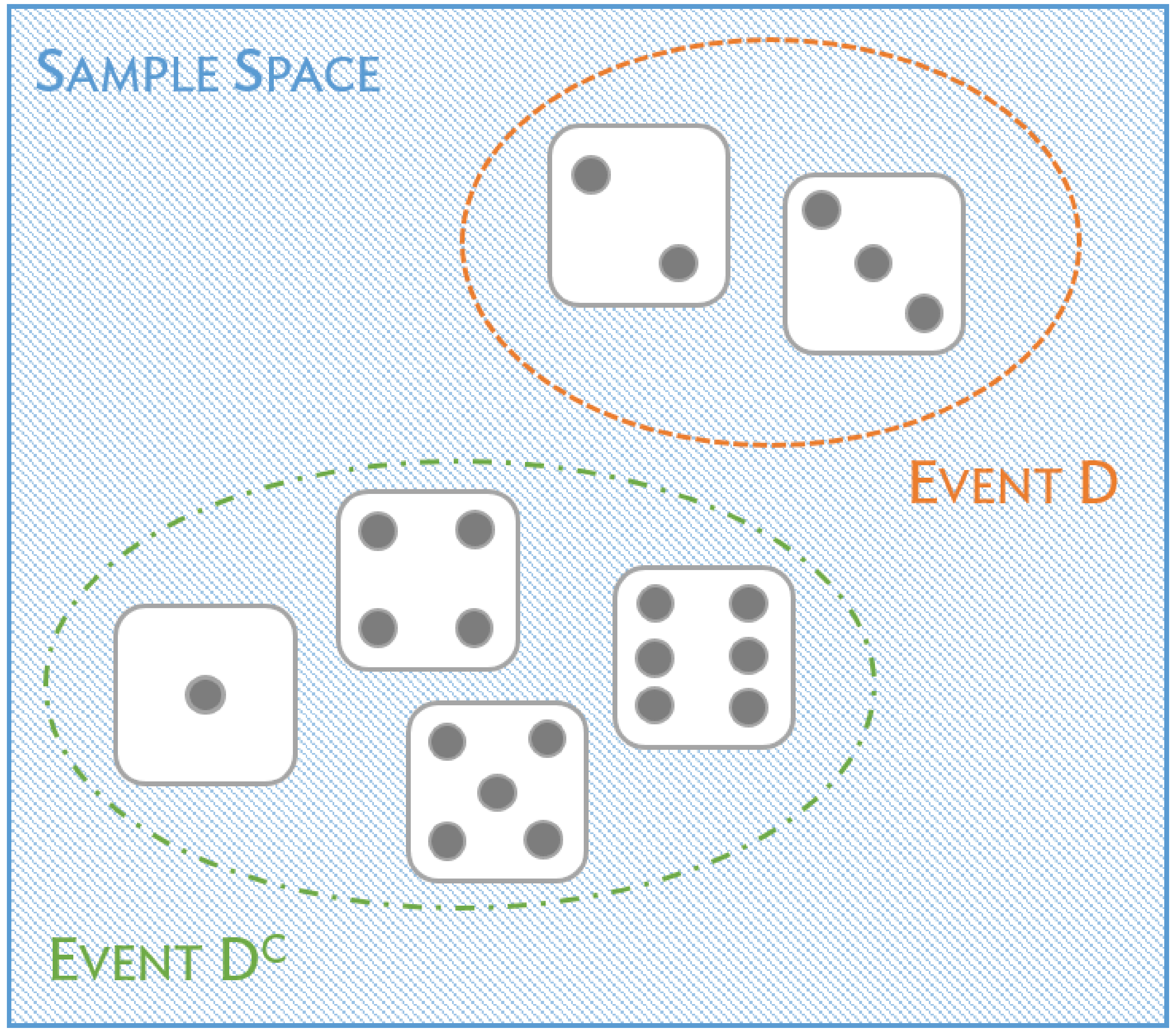
1. The outcomes listed must be disjoint
2. Each probability must be between 0 and 1
3. The probabilities must total to 1



# Complement of an event

We need two math definitions for this:

- **Sample space:** denoted as $S$ is the set of all possible outcomes
- **Complement:** complement of an event, say D, represents all the outcomes in the sample space that are not in D
  - Complement is denoted as $D^c$ or $D'$




**Complement**

The complement of event $A$ is denoted $A^c$, and $A^c$ represents all outcomes not in $A$. $A$ and $A^c$ are mathematically related:

$$P(A) + P(A^c) = 1, \quad \text{i.e.} \quad P(A) = 1 - P(A^c).$$

# Independence

- Two processes are **independent** if knowing the outcome of one provides no information about the outcome of the other
- For example, if we flip two different coins and one lands on heads, what does that tell us about the other coin?

**Multiplication Rule for independent processes**

If $A$ and $B$ represent events from two different and independent processes, then the probability that both $A$ and $B$ occur is given by:

$$P(A \text{ and } B) = P(A)P(B).$$

Similarly, if there are $k$ events $A_1$, ..., $A_k$ from $k$ independent processes, then the probability they all occur is

$$P(A_1)P(A_2)\cdots P(A_k).$$

# Poll Everywhere Question 3

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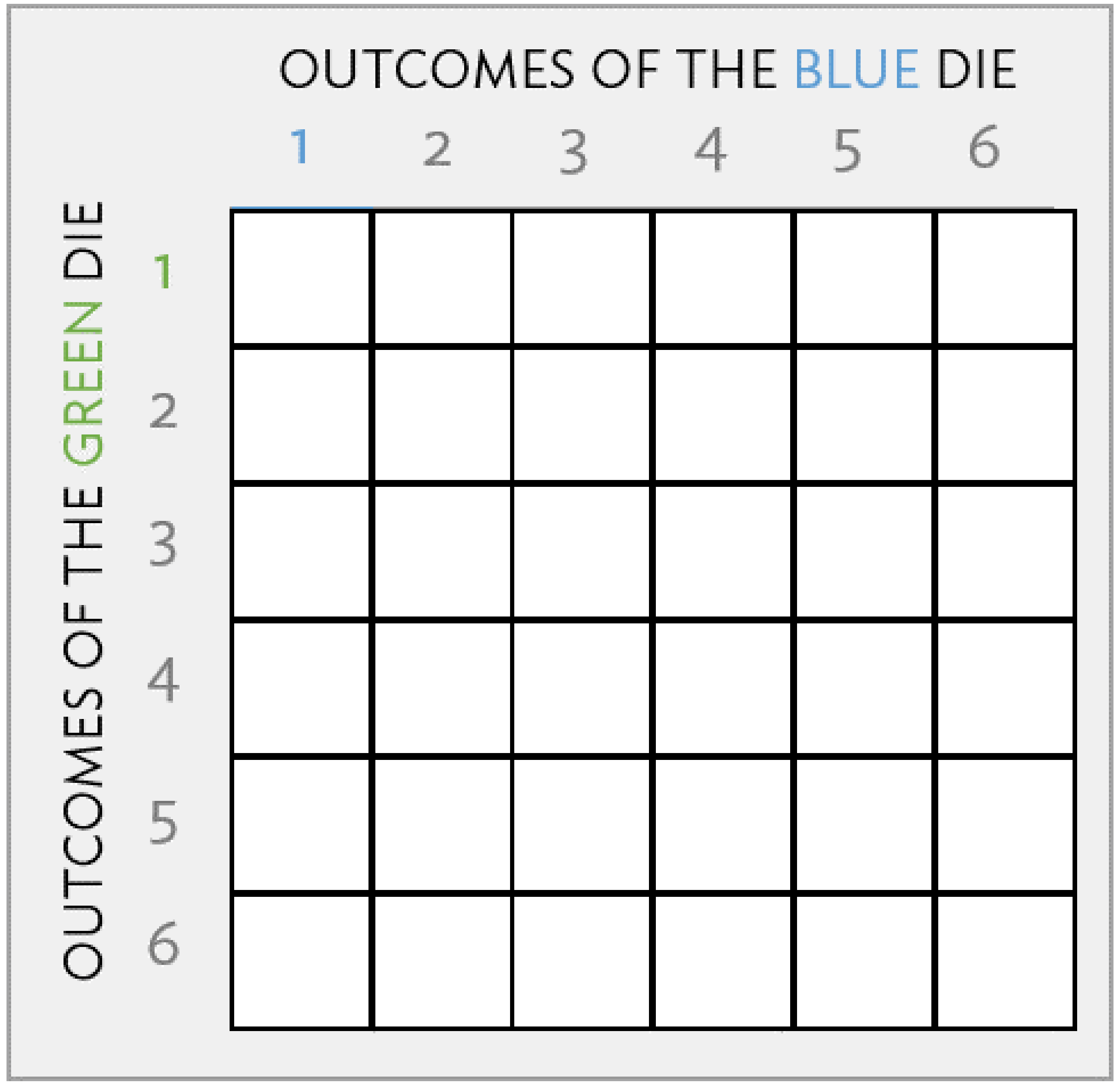
# Example rolling two dice

What is the probability that both dice will be 1?



# General steps for probability word problems

1. Define the events in the problem and make a Venn Diagram
2. Translate the words and numbers into probability statements
3. Translate the question into a probability statement
4. Think about the various definitions and rules of probabilities. Is there a way to define our question's probability statement (in step 3) using the probability statements with assigned values (in step 2)?
5. Plug in the given numbers to calculate the answer!

# Weekly medications

*Hint: Draw a Venn diagram labelling each of the parts to find the probability.*

**Example 3**

If a subject has an

- 80% chance of taking their medication *this* week,
- 70% chance of taking their medication *next* week, and
- 10% chance of *not* taking their medication *either* week,

then find the probability of them taking their medication exactly one of the two weeks.