Example: How accurate is rapid testing for COVID-19? (1/n)

How accurate is rapid testing for COVID-19?

"Based on the results of a clinical study where the iHealth® COVID-19 Antigen Rapid Test was compared to an FDA authorized molecular SARS-CoV-2 test, iHealth® COVID-19 Antigen Rapid Test correctly identified 94.3% of positive specimens and 98.1% of negative specimens." In October 2022, 83.8 people per 100k in Multnomah County with Covid-19.

Suppose you take the iHealth® rapid test.

1. What is the probability of a positive test result?
2. What is the probability of having COVID-19 if you get a positive test result? PPV
3. What is the probability of not having COVID-19 if you get a negative test result? NPV

From the iHealth® website https://ihealthlabs.com/pages/ihealth-covid-19-antigen-rapid-test-details:

Let's apply the steps to our example (1/7)

How accurate is rapid testing for COVID-19?

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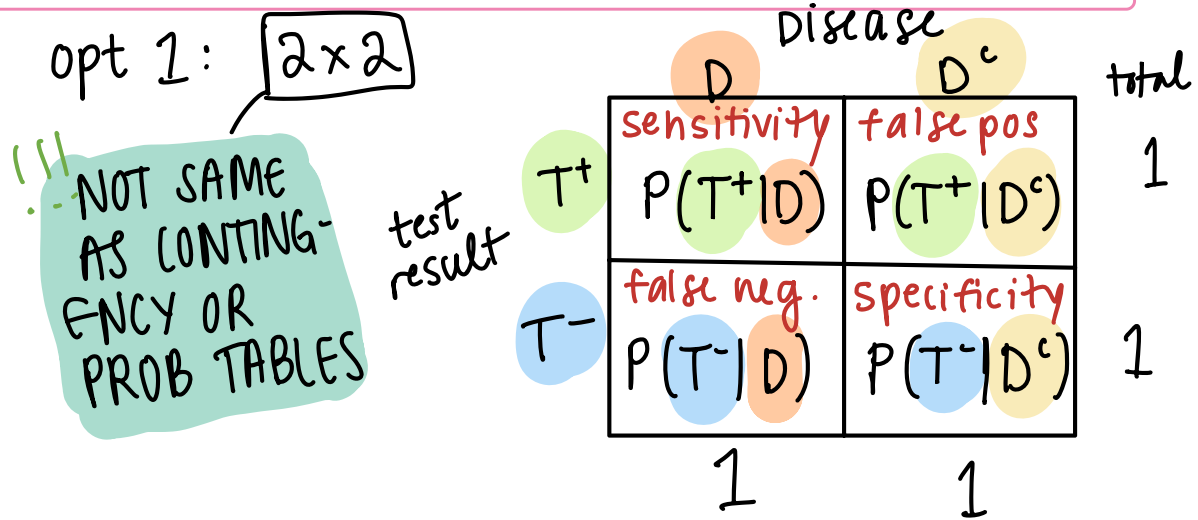
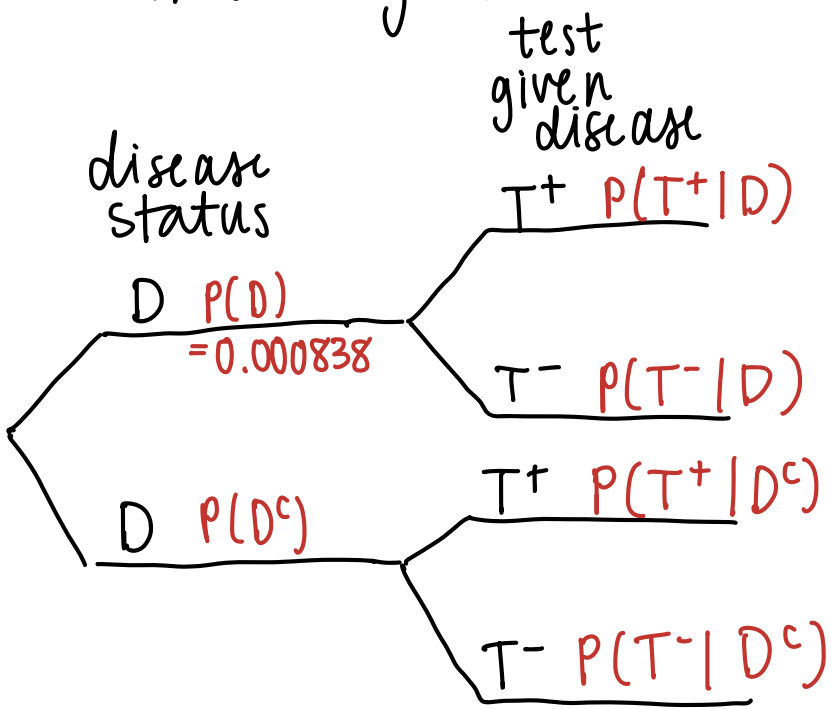
Step 1: Let's define our events of interest

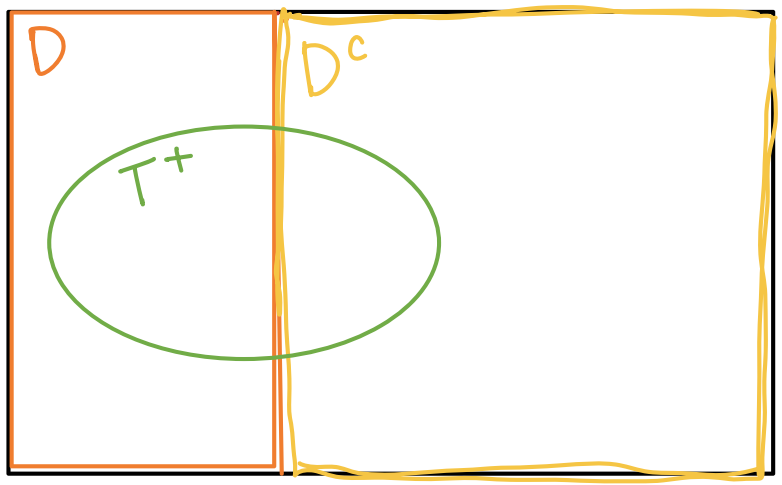
- D = event one has disease (COVID-19)
- D^c = event one does not have disease
- T^+ = event one tests positive for disease
- T^- = event one tests negative for disease



opt 2: tree diagram



opt 3: venn diagram



Let's apply the steps to our example (2/7)

How accurate is rapid testing for COVID-19?

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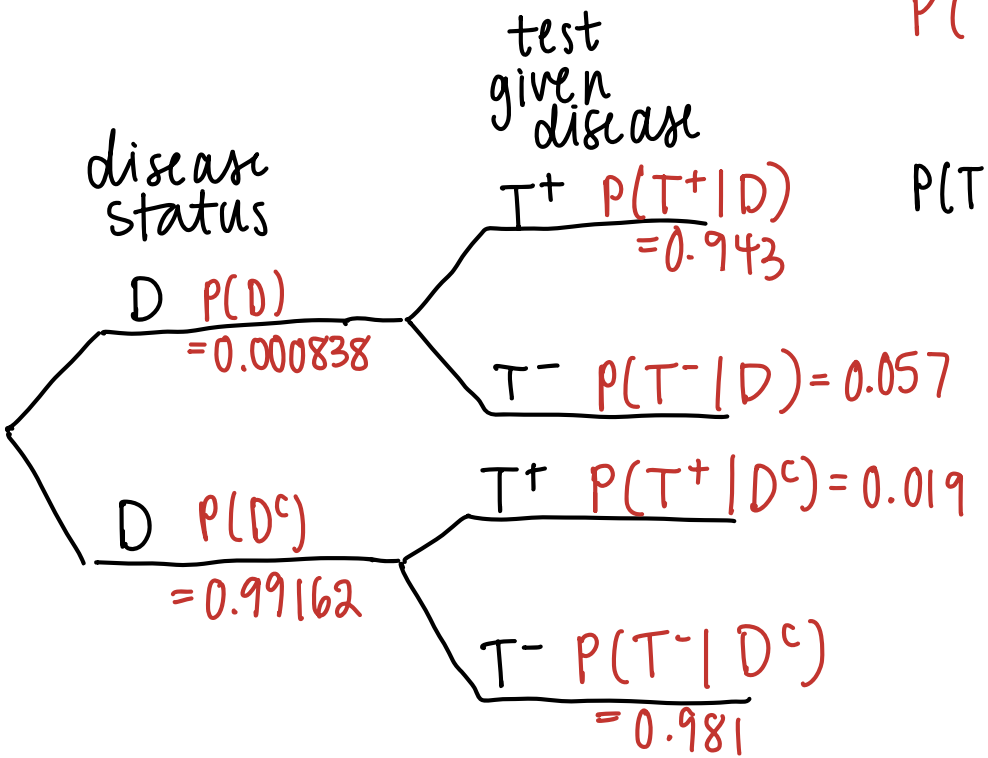
Step 2: Translate given information into mathematical notation

- Test correctly gives a positive result 94.3% of the time: $P(T^+|D) = 0.943$
- Test correctly gives a negative result 98.1% of the time: $P(T^-|D^c) = 0.981$
- 83.8 people per 100k in Multnomah County with Covid-19: $P(D) = 0.000838$

$P(D^c) = 1 - P(D)$

$P(T^+|D) + P(T^-|D) = 1$ muddy pt #6



$P(T^+ \cap D) = P(D)P(T^+|D) = 0.000838 \cdot 0.943$

Let's apply the steps to our example (3/7)

How accurate is rapid testing for COVID-19?

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Step 3: Translate the question into a probability statement

1. What is the probability of a positive test result? $P(T^+)$

2. What is the probability of having COVID-19 if you get a positive test result? $P(D \mid T^+)$

3. What is the probability of not having COVID-19 if you get a negative test result? $P(D^c \mid T^-)$

use Bayes' thm here

Law of Total Probability: $P(T^+) = P(T^+ \cap D) + P(T^+ \cap D^c)$

by Multiplication Rule. $P(T^+|D)P(D) + P(T^+|D^c)P(D^c)$

Let's apply the steps to our example (4/7)

How accurate is rapid testing for COVID-19?

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Step 4: Define our question's probability statement using the probability statements with assigned values

1. $P(T^+) =$ $P(T^+ \mid D)P(D) + P(T^+ \mid D^c)P(D^c)$

$= 0.943(0.000838) + (1-0.981)(1-0.000838)$

$= 0.01977$

The probability of a positive test is 0.01977.

Lesson 4 Slides

Let's apply the steps to our example (5/7)

How accurate is rapid testing for COVID-19?

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Step 4: Define our question's probability statement using the probability statements with assigned values

by multiplication rule

2. $P(D|T^+) = \dfrac{P(D \cap T^+)}{P(T^+)} = \dfrac{P(T^+|D)P(D)}{P(T^+)}$

ALSO BY BAYES

$P(D|T^+) = \dfrac{P(T^+|D)P(D)}{P(T^+)}$

know this from above (Law of total prob)

$= \dfrac{(0.943)(0.000838)}{0.01977}$

writing this but calculating w/ full value

$= 0.03996$

The probability that someone has COVID-19 if they tested positive is 0.03996

PPV

Let's apply the steps to our example (6/7)

How accurate is rapid testing for COVID-19?

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Step 4: Define our question's probability statement using the probability statements with assigned values

NPV

3. $P(D^c|T^-) =$

Bayes

$$= \frac{P(T^-|D^c)P(D^c)}{P(T^-)}$$

$$= \frac{P(T^-|D^c)P(D^c)}{P(T^-|D^c)P(D^c) + P(T^-|D)P(D)}$$

law of total prob

$P(T^- \cap D^c)$ $\quad$ $P(T^- \cap D)$

$$= \frac{(0.981)(1-0.000838)}{(0.981)(1-0.000838)+(1-0.943)(0.000838)}$$

$$= 0.99995$$

$P(T^-|D) = 1 - P(T^+|D)$

muddy pt #6

The probability that someone does NOT have Covid given a negative test result is 0.99995

Let's apply the steps to our example (7/7)

How accurate is rapid testing for COVID-19?

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Step 5: Calculate answer

Lesson 4 Slides