# Lesson 9: Variability in estimates

TB sections 4.1

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Artwork by @allison_horst

# Learning Objectives

1. Illustrate how information from several samples are connected to the population and to the sampling distribution
2. Understand how the sampling distribution of the sample means relates to a sample and the population distribution
3. Apply the Central Limit Theorem to approximate the sampling distribution of the sample mean

# Where are we?

**Data**
- Collecting data
- Categorical vs. Numeric
- Summary statistics
- Data visualization

**Probability**
- Probability rules
- Independence, conditional
- Random variables and probability distributions
- Linear combinations
- Binomial, Normal, and Poisson

**Sampling Variability, and Statistical Inference**
- Sampling distributions
- Central Limit Theorem
- Confidence Intervals
- Hypothesis tests

**Inference for continuous data/outcomes**
- One sample t-test
- 3+ independent samples
- Simple linear regression / correlation
- 2 sample tests: paired and independent
- Power and sample size
- Non-parametric tests

**Inference for categorical data/outcomes**
- One proportion test
- Fisher's exact test
- Non-parametric tests
- Chi-squared test
- 2 proportion test
- Power and sample size

**R**
- Basics
- Reproducibility
- Quarto
- Packages
- Data visualization
- Data wrangling
- R Projects
- …

# From Lesson 1: Population vs. sample

**(Target) Population**

- Group of interest being studied
- Group from which the sample is selected
  - Studies often have *inclusion* and/or *exclusion* criteria
- Almost always too expensive or logistically impossible to collect data for every case in a population

**Sample**

- Group on which data are collected
- Often a **small subset** of the population
- Easier to collect data on
- If we do it right, we might be able to answer our question about the target population

- Goal is to get a **representative** sample of the population: the characteristics of the sample are similar to the characteristics of the population

# Why sample statistics?

- When we want to estimate features of the population
  - We can use corresponding summary statistics calculated from our sample
  - Often called **point estimates** or **sample statistics**

- Much easier to measure statistics from our sample (Lesson 1)
  - However, statistics from our sample are not exactly the same as the population measurements that we're aiming for
  - We call the population measurements **population parameters**

- So we need to start by distinguishing between the population parameters and sample statistics

# Population parameters vs. sample statistics

## Population parameter

- Mean: $\mu$ ("mu")
- Standard deviation: $\sigma$ ("sigma")
- Variance: $\sigma^2$

- Proportion: $p, \pi$ ("pi")

- Correlation

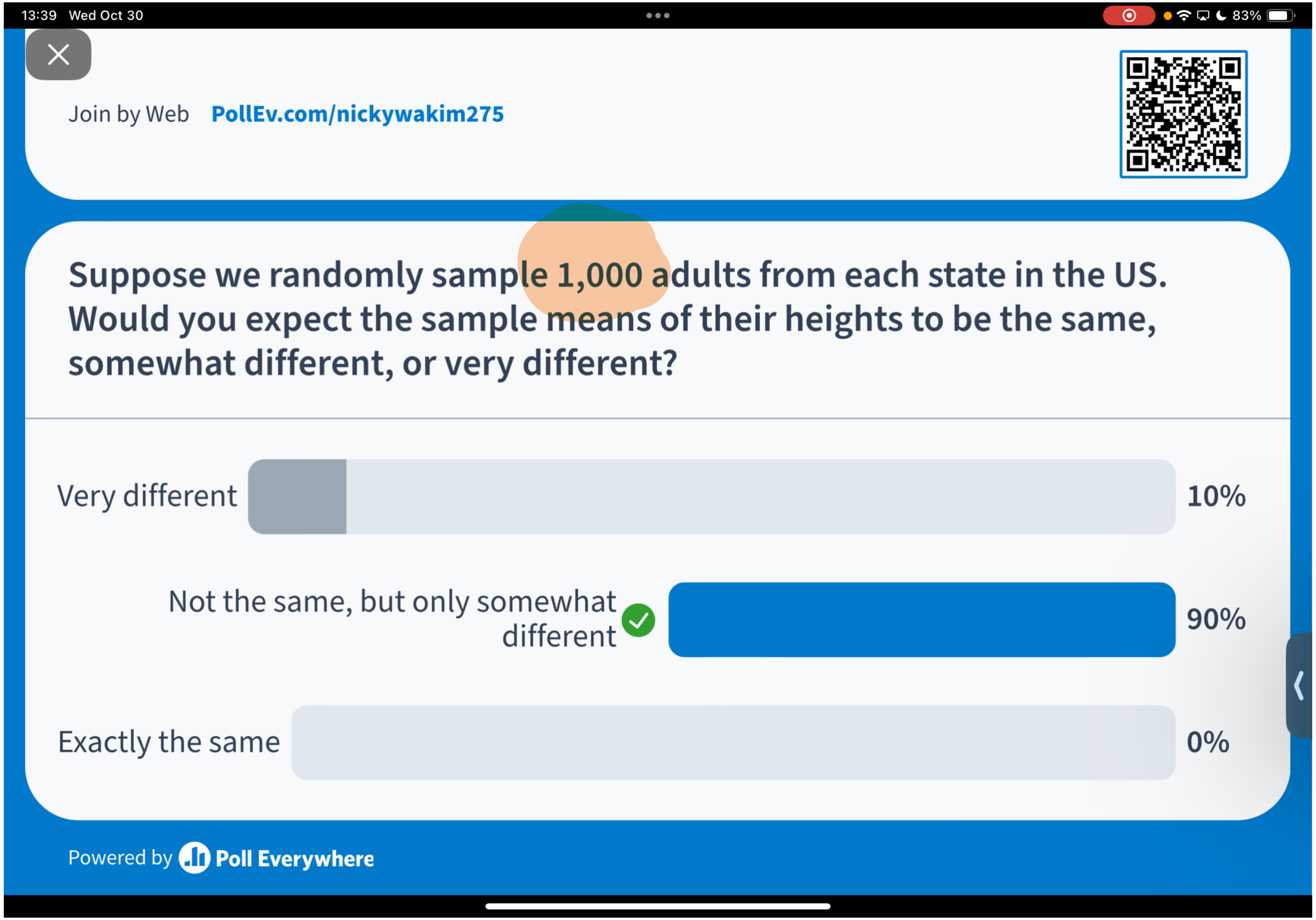
## Sample statistic (point estimate)

- Sample mean: $\overline{x}$ x-bar
- Sample standard deviation: $s$
- Sample variance: $s^2$

- Sample proportion: $\hat{p}$ ("p-hat")

- Sample correlation coefficient: $r$

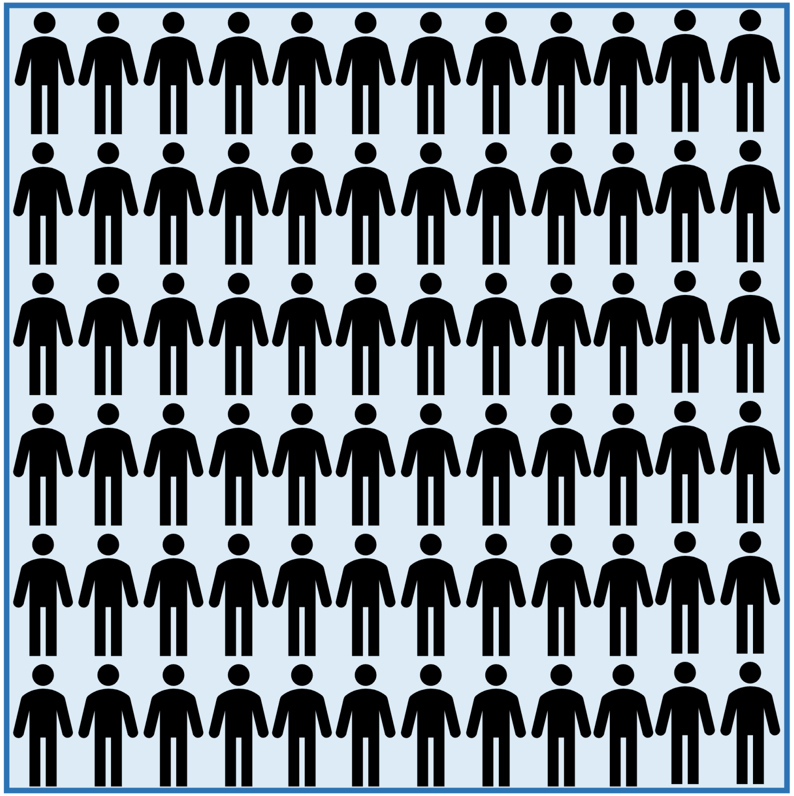
# Poll Everywhere Question 1



# Learning Objectives

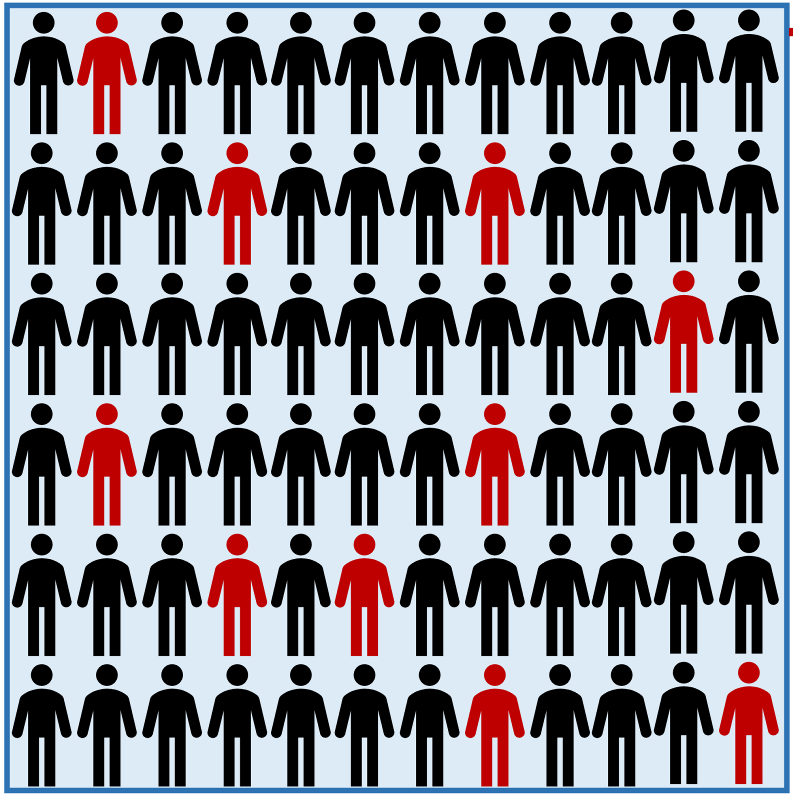
1. Illustrate how information from several samples are connected to the population and to the sampling distribution
2. Understand how the sampling distribution of the sample means relates to a sample and the population distribution
3. Apply the Central Limit Theorem to approximate the sampling distribution of the sample mean

# Population



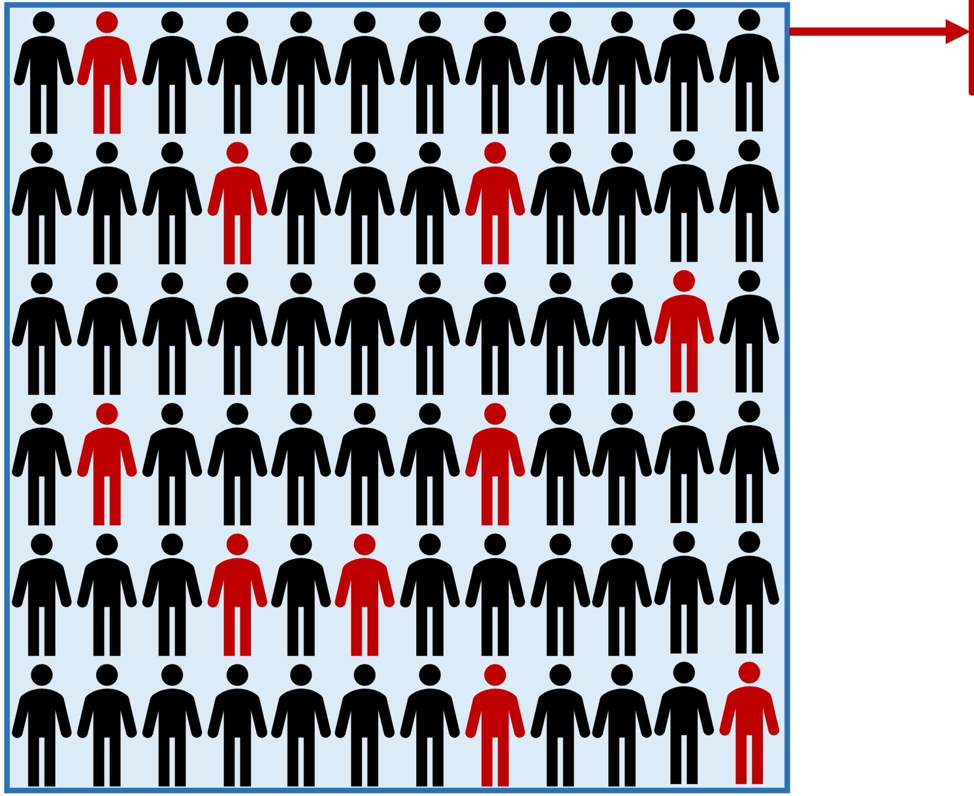
Example to facilitate our thinking:
**Population height**

# Take one sample



Sample of red people
Mean: $\bar{x}$, SD: $s$

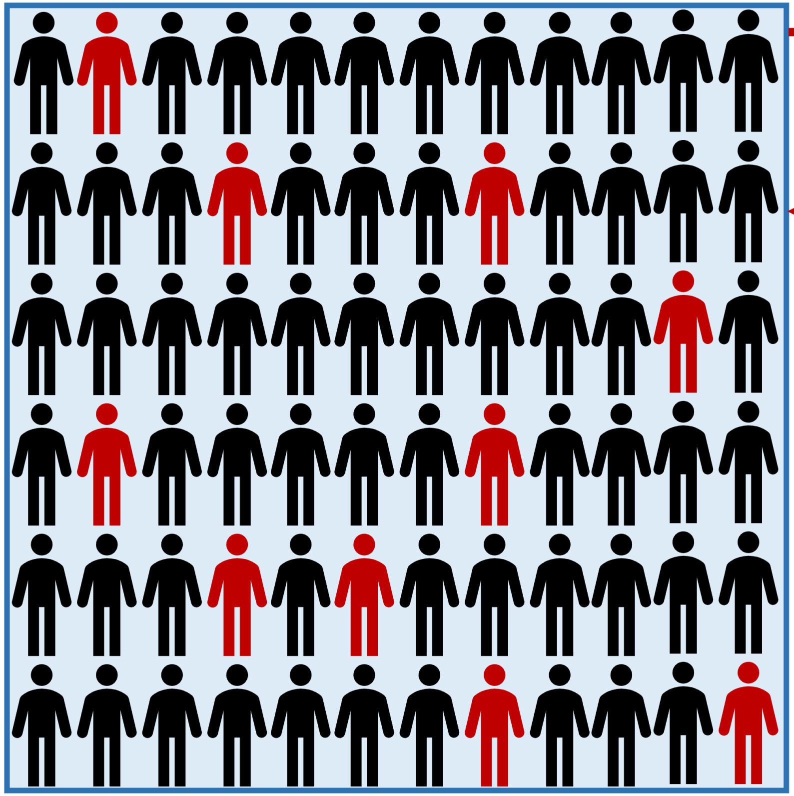
# Take one sample



Sample of red people
Mean: $\bar{x}$, SD: $s$

Heights of 10 people:
67 inches
57 inches
70 inches
61 inches
69 inches
68 inches
62 inches
75 inches
65 inches
59 inches

# Take one sample



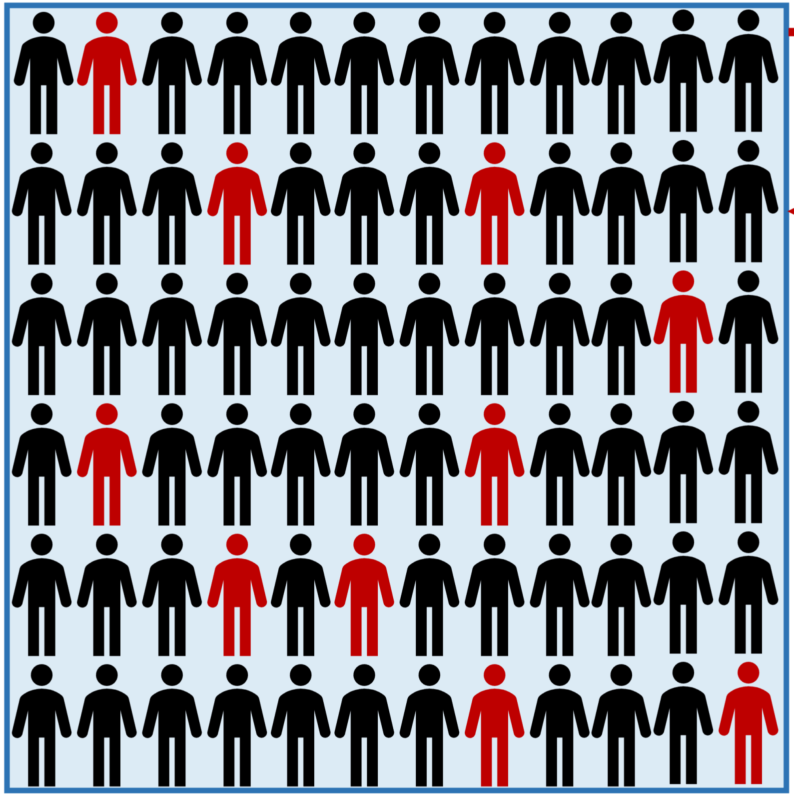
Sample of red people
Mean: $\bar{x}$, SD: $s$

$\bar{x}$ = 65.3
$s$ = 5.6

From our red people sample, this is our **best estimate** of the population's average height and standard deviation

When we are researching, this is usually **the end of our data collection process**!

# Take one sample



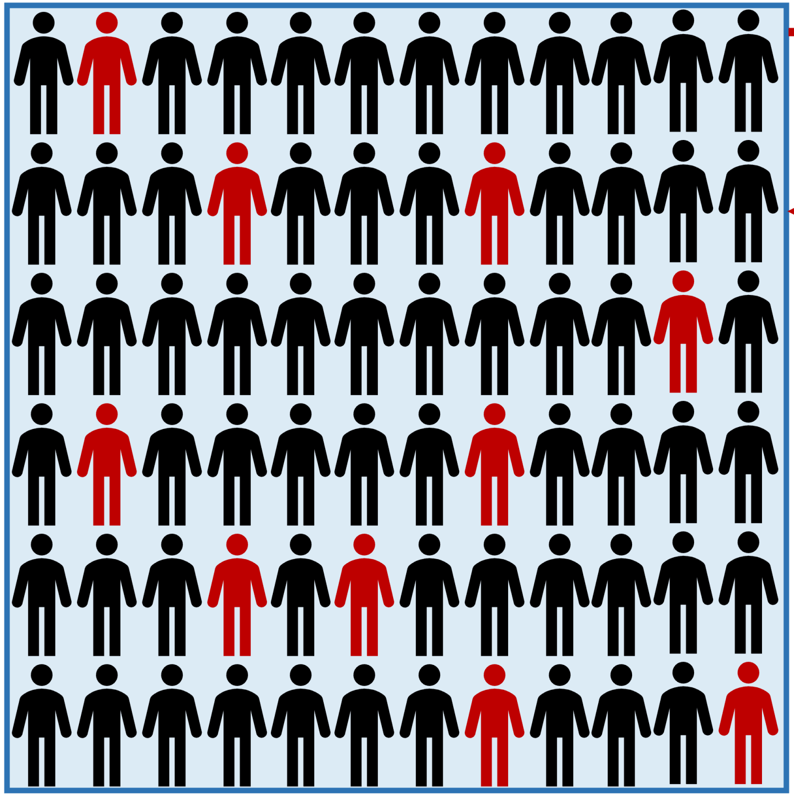
Sample of red people
Mean: $\bar{x}$, SD: $s$

$\bar{x}$ = 65.3
$s$ = 5.6

Even though this is the best estimate based on our sample, it **may not truly capture the population.**

Can we measure how well our sample captures the population?

# Take one sample



Sample of red people
Mean: $\bar{x}$, SD: $s$

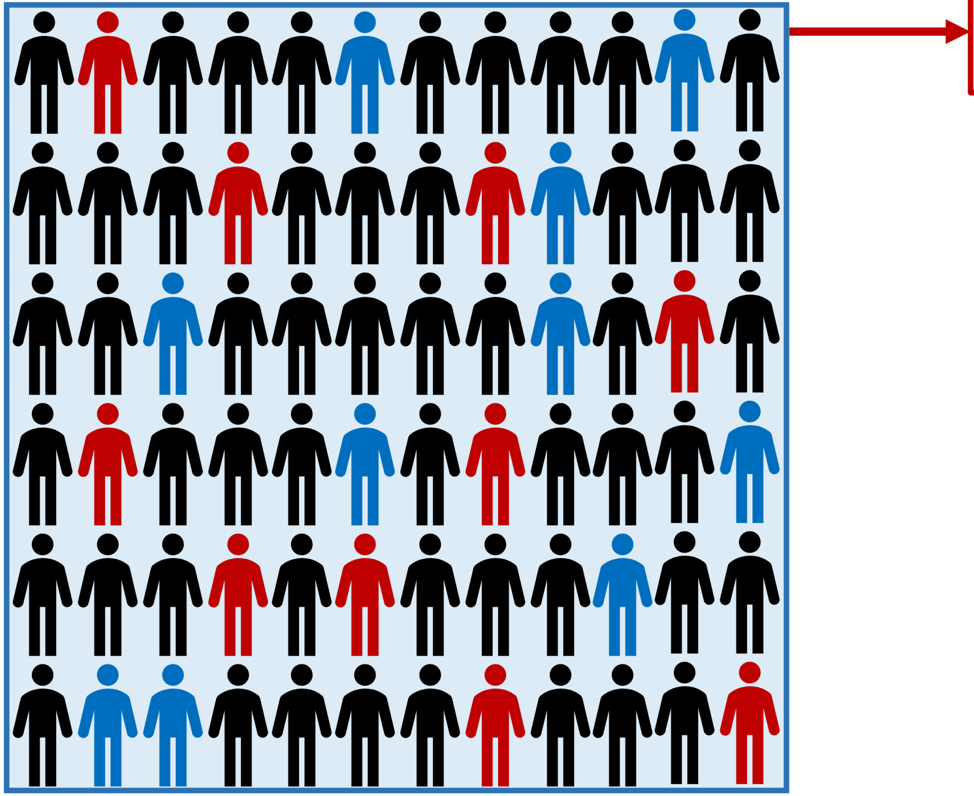
$\bar{x}$ = 65.3
$s$ = 5.6

Even though this is the best estimate based on our sample, it **may not truly capture the population.**

Can we measure how well our sample captures the population?

**BUILD A SAMPLING DISTRIBUTION**

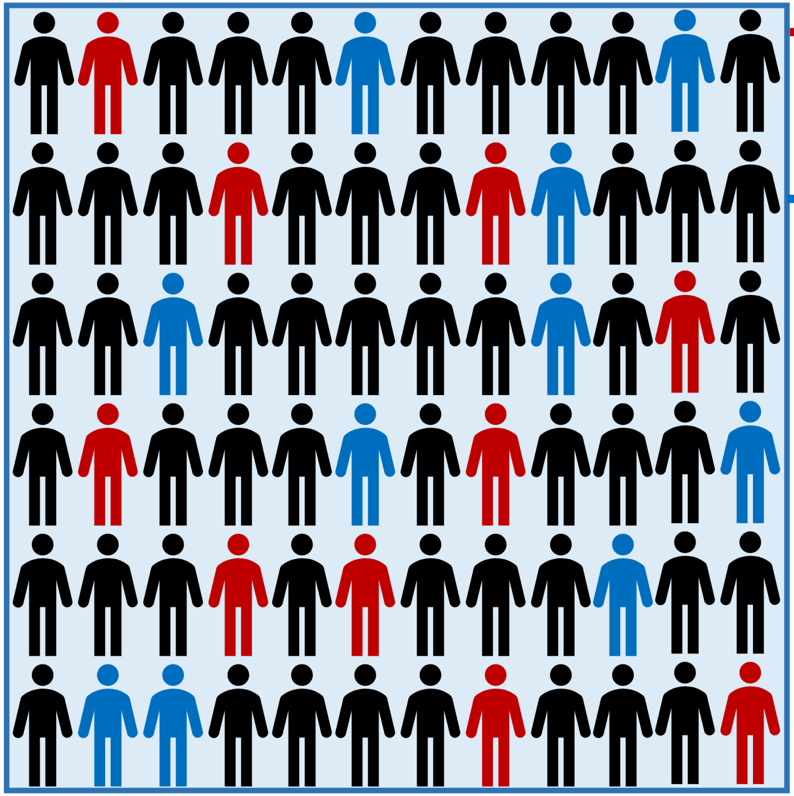
# Take another sample



Sample of red people
Mean: $\bar{x}$, SD: $s$

$\bar{x}$ = 65.3
$s$ = 5.6

# Take another sample

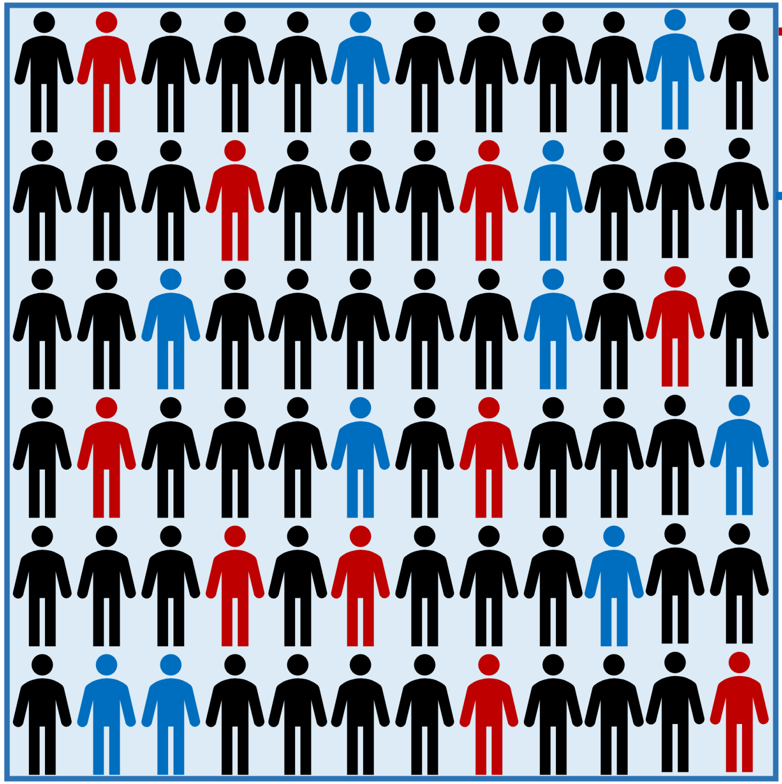


Sample of red people
Mean: $\bar{x}$, SD: $s$

$\bar{x}$ = 65.3
$s$ = 5.6

Sample of blue people
Mean: $\bar{x}$, SD: $s$

# Take another sample



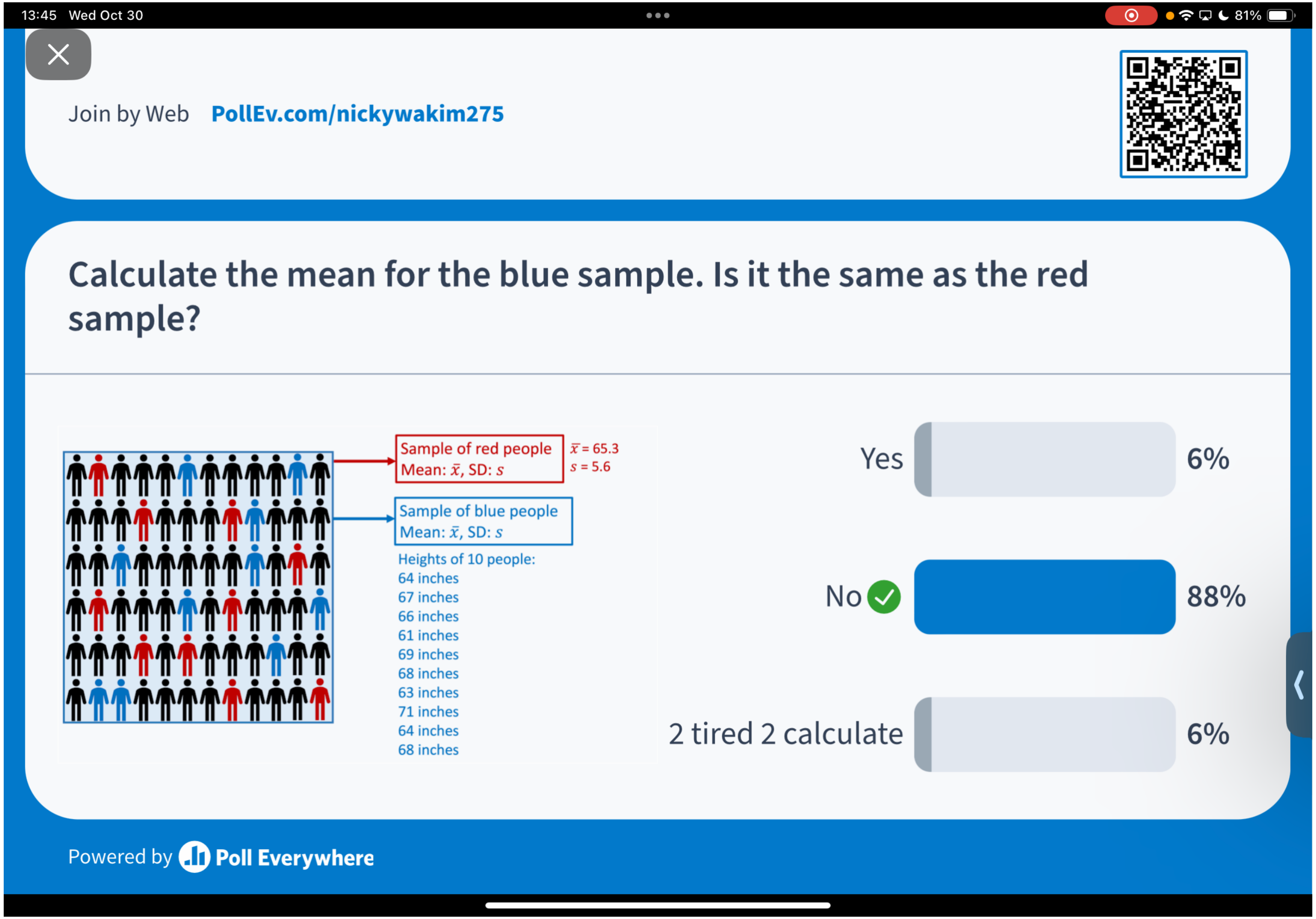
Sample of red people
Mean: $\bar{x}$, SD: $s$

$\bar{x} = 65.3$
$s = 5.6$

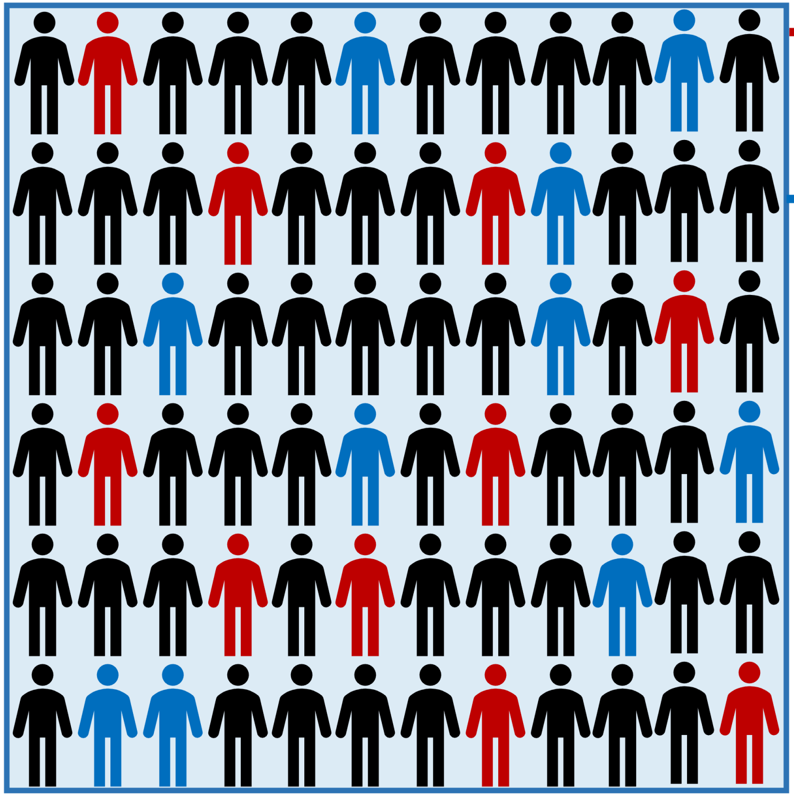
Sample of blue people
Mean: $\bar{x}$, SD: $s$

Heights of 10 people:
64 inches
67 inches
66 inches
61 inches
69 inches
68 inches
63 inches
71 inches
64 inches
68 inches

# Poll Everywhere Question 2



# Take another sample



Sample of red people
Mean: $\bar{x}$, SD: $s$

$\bar{x}$ = 65.3
$s$ = 5.6

Sample of blue people
Mean: $\bar{x}$, SD: $s$

$\bar{x}$ = 66.1 ←
$s$ = 3.1

Are these two sample means the same?

# Take several samples

Sample of red people
Mean: $\bar{x}$, SD: $s$

Sample of blue people
Mean: $\bar{x}$, SD: $s$

Sample of orange people
Mean: $\bar{x}$, SD: $s$

Sample of green people
Mean: $\bar{x}$, SD: $s$

# Difference between samples?

From our **red sample alone**, we don't know how well it captures the population.

If we had access to many samples, we'd get a better picture of the population AND how the red sample relates to the population.

# Learning Objectives

1. Illustrate how information from several samples are connected to the population and to the sampling distribution

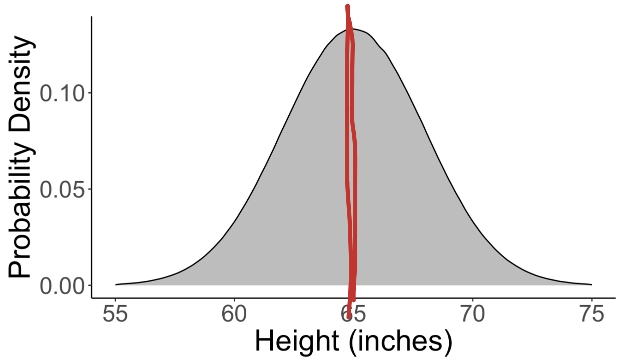
$\bar{x}$'s

2. Understand how the sampling distribution of the sample means relates to a sample and the population distribution

3. Apply the Central Limit Theorem to approximate the sampling distribution of the sample mean

# More concrete example with height (1/3)

Variation in population ($\sigma$):

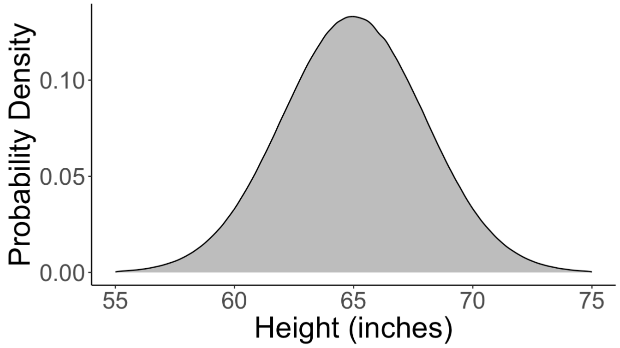



$$\mu = 65 \text{ inches}$$

$$\sigma = 3 \text{ inches}$$

# More concrete example with height (2/3)

Variation in population ($\sigma$):




$$\mu = 65 \text{ inches}$$

$$\sigma = 3 \text{ inches}$$

Variation within samples ($s$):

1. Sample 50 people
   $\bar{x} = 65.1, s = 2.8$
2. Sample 50 people
   $\bar{x} = 64.7, s = 3.1$
3. Sample 50 people
   $\bar{x} = 64.7, s = 2.5$
4. Sample 50 people
   $\bar{x} = 66.1, s = 3.4$
5. Sample 50 people
   $\bar{x} = 65.3, s = 2.9$

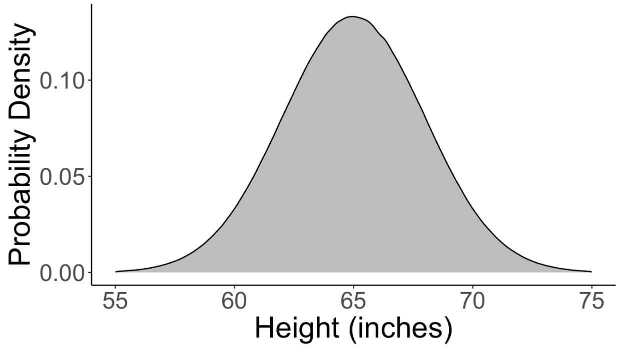
...

1000. Sample 50 people
   $\bar{x} = 64.9, s = 3.2$

# More concrete example with height (3/3)

sampling distribution

Variation in population ($\sigma$):




$$\mu = 65 \text{ inches}$$

$$\sigma = 3 \text{ inches}$$

Variation within samples ($s$):

1 Sample 50 people
$\bar{x} = 65.1, s = 2.8$

2 Sample 50 people
$\bar{x} = 64.7, s = 3.1$

3 Sample 50 people
$\bar{x} = 64.7, s = 2.5$

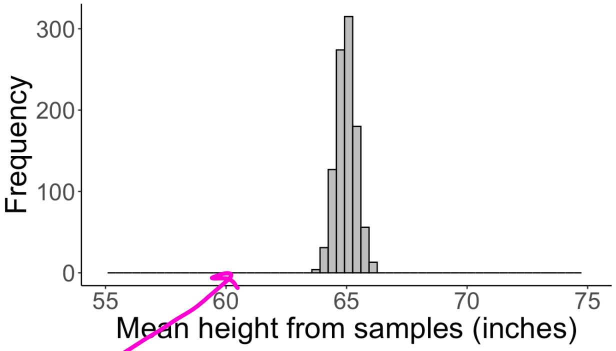
4 Sample 50 people
$\bar{x} = 66.1, s = 3.4$

5 Sample 50 people
$\bar{x} = 65.3, s = 2.9$

...

1000 Sample 50 people
$\bar{x} = 64.9, s = 3.2$

Variation between samples ($SE$):



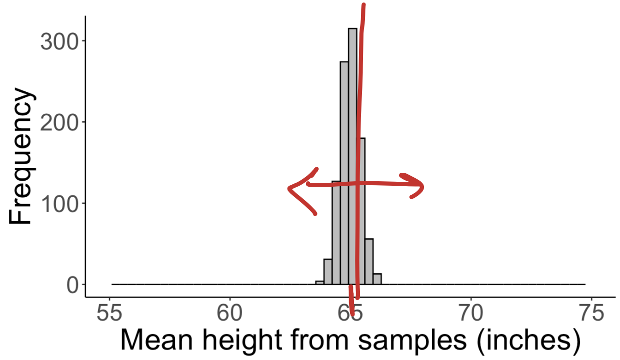

$$\mu_{\overline{X}} = 64.975 \text{ inches}$$

$$SE = 0.414 \text{ inches}$$

# Sampling Distribution of Sample Means

- The **sampling distribution** is the distribution of sample means calculated from repeated random samples of *the same size* from the same population
- It is useful to think of a **particular sample statistic** as being **drawn from a sampling distribution**
  - So the red sample with $\overline{x} = 65.1$ is **just one sample mean** in the **sampling distribution**

Variation between samples ($SE$):

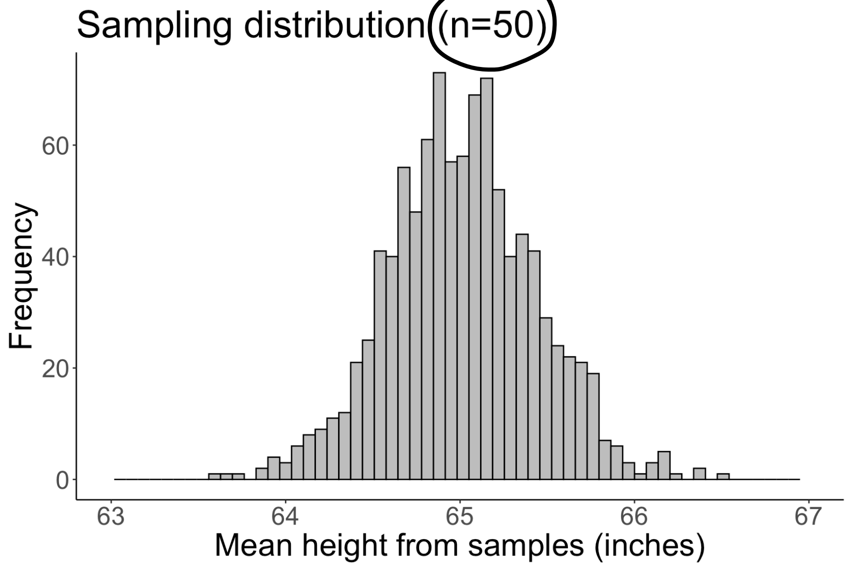


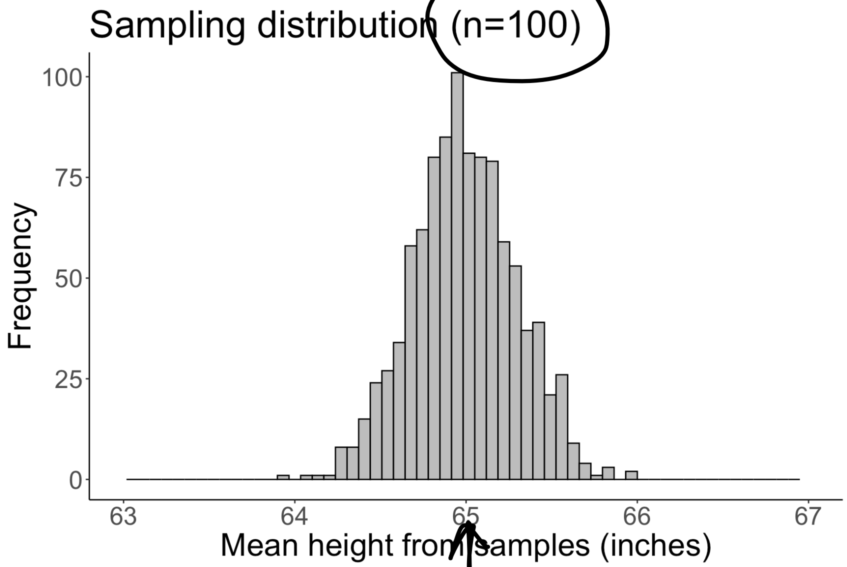

$$\mu_{\overline{X}} = 64.975 \text{ inches}$$

$$SE = 0.414 \text{ inches}$$

# For following Poll Everywhere Question

How are the center, shape, and spread similar and/or different?



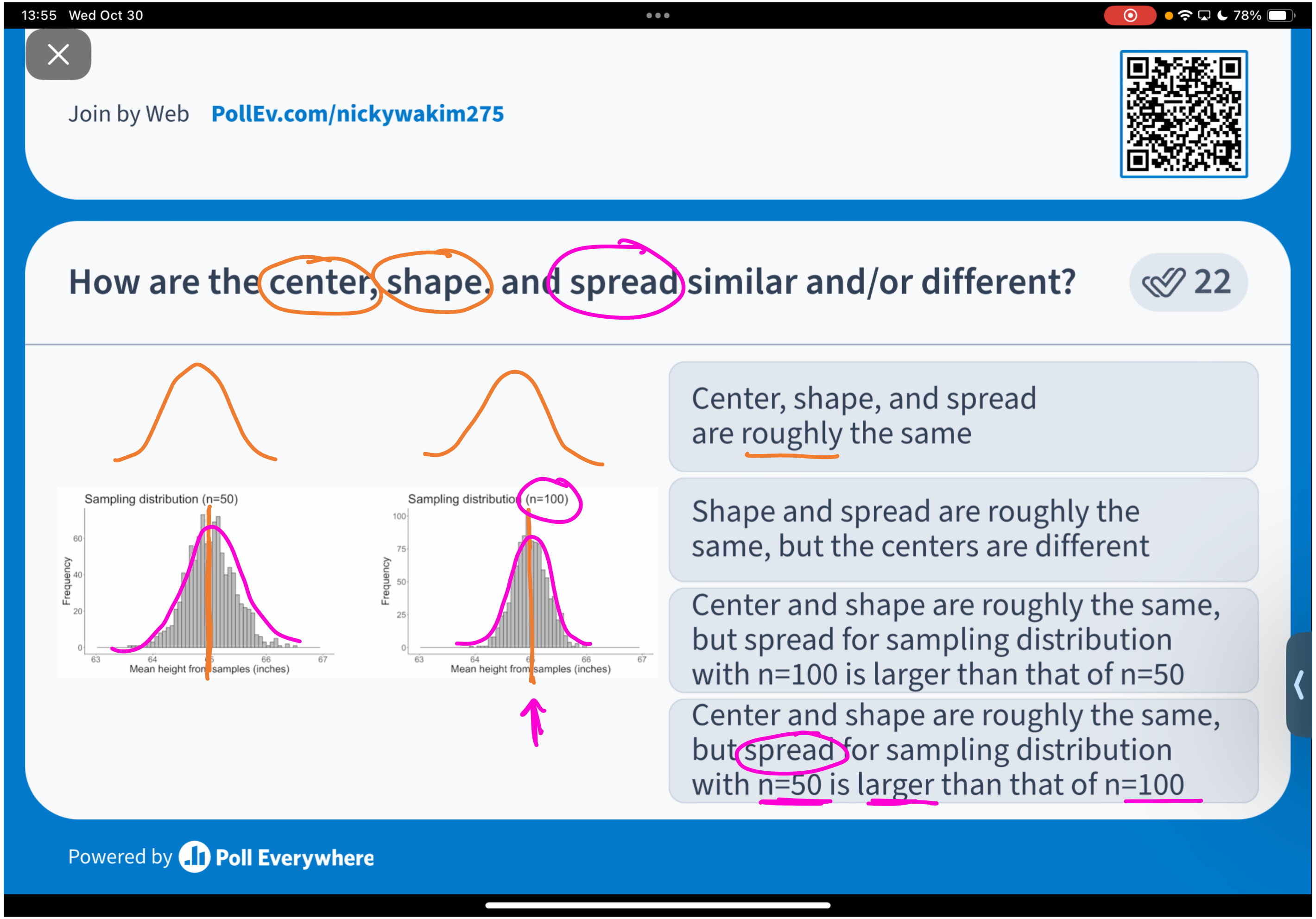




$$SE = \frac{\sigma}{\sqrt{n}} = \frac{\sigma}{\sqrt{50}}$$

$$\frac{\sigma}{\sqrt{50}} > \frac{\sigma}{\sqrt{100}}$$

$$SE = \frac{\sigma}{\sqrt{n}} = \frac{\sigma}{\sqrt{100}}$$

# Poll Everywhere Question 3



# Okay, but in real life we only have one sample...?

- In a study, conclusions about a population parameter **must be drawn from the data collected from a single sample**

- The sampling distribution of $\widetilde{X}$ is a **theoretical concept**
  - Obtaining repeated samples by conducting a study many times is not possible

- **Not feasible** to calculate the population mean $\mu$ by finding the mean of the sampling distribution for $\overline{X}$

- In the next lesson on confidence intervals, we'll see what kind of statements we can make about the population mean from a single sample

# Learning Objectives

1. Illustrate how information from several samples are connected to the population and to the sampling distribution
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3. Apply the Central Limit Theorem to approximate the sampling distribution of the sample mean

# The Central Limit Theorem (CLT)

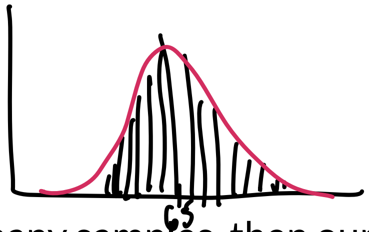
n = 50
sampled 50 ppl's heights

- If a sample consists of **at least 30 independent observations**, then the sampling distribution of the sample mean is **approximated by a normal model**

@ least 30 in EACH sample

↳ $\bar{x}$'s for all our samples of 50 ppl

- Aka, for **"large" sample sizes** ( $n \geq 30$ ),
  - The sampling distribution of the sample mean can be approximated by a **normal distribution**,with
    - Mean equal to the population mean value $\mu$
    - Standard deviation $\frac{\sigma}{\sqrt{n}}$
- This is **regardless of the original sample is from a different distribution**
  - For example, if we count the number of heads in 50 coin flips, and do this for many samples, then our sampling distribution will be Normally distributed around $n \cdot p = 50 \cdot 0.5 = 25$




standard error : quick way to say "std deviation of the sampling distribution of the sample means"

# Other cases for normal approximation

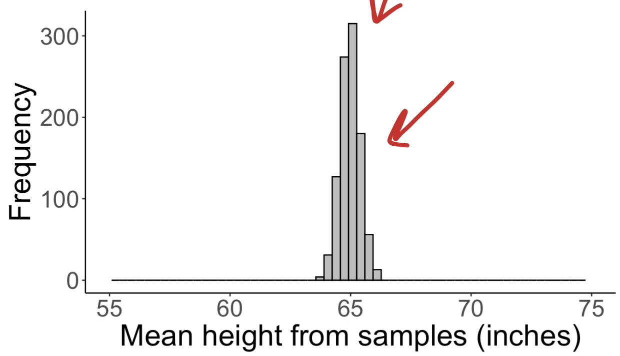
- For **small sample sizes**, if the **population is known to be normally distributed**, then
  - The sampling distribution of the sample mean is a **normal distribution**, with
    - Mean equal to the population mean value $\mu$, and
    - Standard deviation $\frac{\sigma}{\sqrt{n}}$
- Not technically the Central Limit Theorem, but sampling distribution approximated using same Normal distribution

# Sampling Distribution of Sample Means (with the CLT)

★ why 30 w/ sims

- The **sampling distribution** is the distribution of sample means calculated from repeated random samples of *the same size* from the same population
- It is useful to think of a **particular sample statistic** as being **drawn from a sampling distribution**
  - So the red sample with $\overline{x} = 65.1$ is **just one sample mean** in the **sampling distribution**

Variation between samples ($SE$):




$\mu_{\overline{X}} = 64.99$ inches

$SE = 0.304$ inches

**With CLT and $\overline{X}$ as the RV for the sampling distribution**

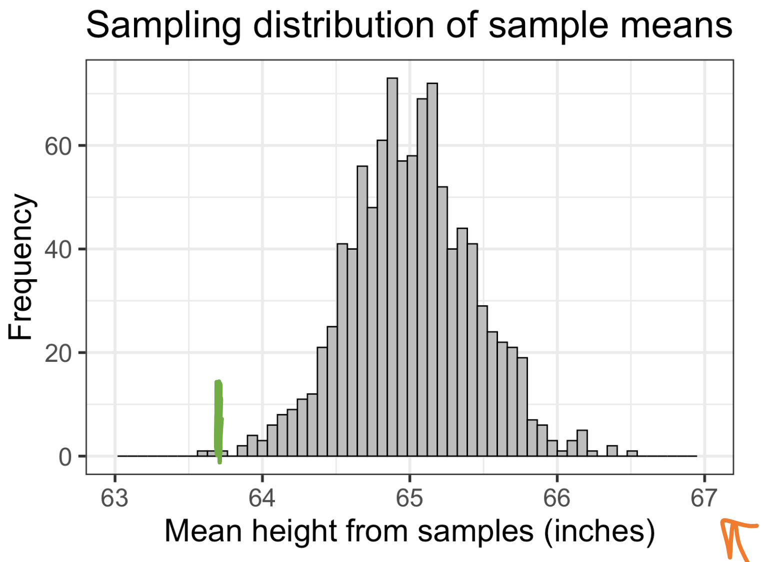
- **Theoretically** (using only population values):
  $\overline{X} \sim \text{Normal}\left(\mu_{\overline{X}} = \mu, \sigma_{\overline{X}} = SE = \frac{\sigma}{\sqrt{n}}\right)$
- **In real use** (using sample values for SE):
  $\overline{X} \sim \text{Normal}\left(\mu_{\overline{X}} = \mu, \sigma_{\overline{X}} = SE = \frac{s}{\sqrt{n}}\right)$

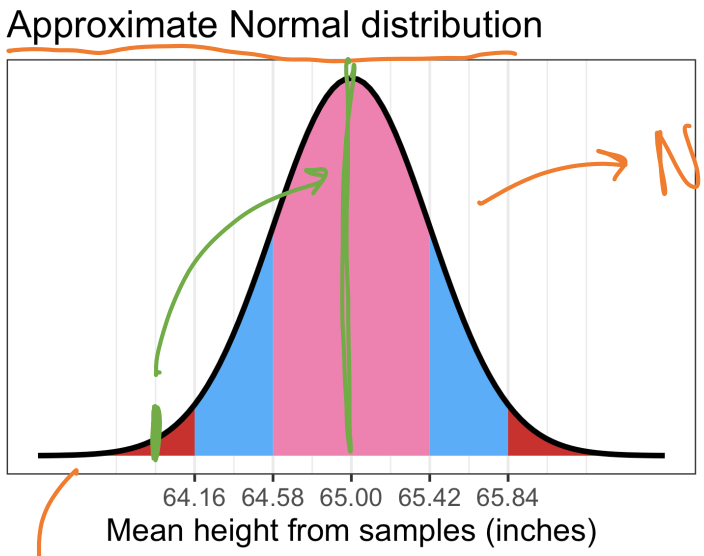
use std dev from sample to estimate standard error

sample mean RV (random variable)

# Let’s apply the CLT to our sampling distribution when n = 50 (1/2)

CLT tells us that we can model the sampling distribution of mean heights using a normal distribution:




of this sampling dist.

normal approximation

$N(\mu, \frac{\sigma}{\sqrt{50}})$

# Let's apply the CLT to our sampling distribution when n = 50 (2/2)

Mean and SD of **population**:

$$\mu = 65 \text{ inches}, \sigma = 3 \text{ inches}$$

From the CLT, we can figure out the **theoretical** mean and standard deviation of our sampling distribution:

$$\mu = 65 \text{ inches}$$

$$SE = \frac{\sigma}{\sqrt{n}} \text{ inches} = \frac{3}{\sqrt{50}} \text{ inches} = 0.424 \text{ inches}$$

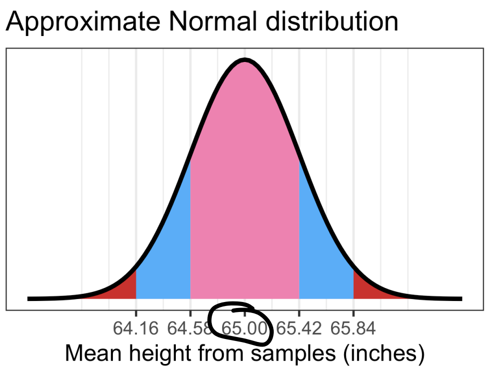
I simulated the data, so I can calculate mean and SE of the sampling distribution:

```
(sample_mean = mean(means50$means))
```

```
[1] 65.01157
```

```
(sample_se = sd(means50$means))
```

```
[1] 0.4254565
```




[ & red sample ]

$\bar{x} = 65.1$ (mean of 50 ppl) $s = 2.8$

$SE = \frac{s}{\sqrt{n}} = \frac{2.8}{\sqrt{50}} = 0.396$

# Applying the CLT (1/2)

don't know how height is distributed

**Example 1**

For a random sample of 100 people, what is the probability that their mean height is greater than 65 inches? We happen to know the population mean is 64 inches and population standard deviation is 4 inches.

1. Make sure that the number of individuals in the sample is greater than 30: $100 > 30$, so we can use the CLT
2. Find the mean and standard error for our sampling distribution:

$$\mu_{\overline{X}} = 64$$

$$SE = \frac{\sigma}{\sqrt{n}} = \frac{4}{\sqrt{100}} = 0.4 \text{ inches}$$

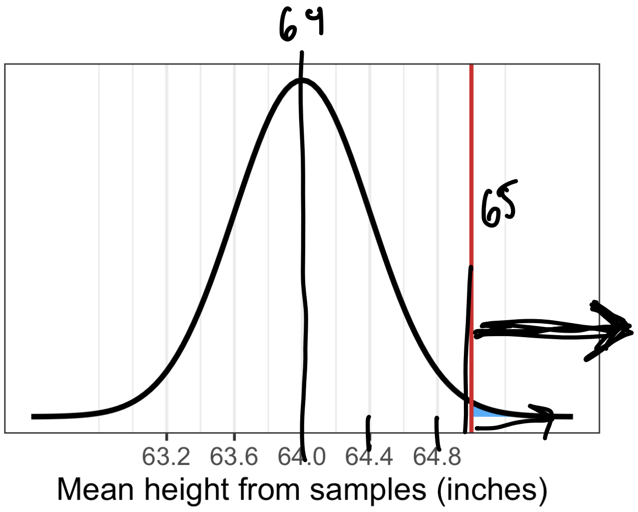
$$\overline{X} \sim \text{Normal}(64, 0.4)$$

# Applying the CLT (2/2)

**Example 1**

For a random sample of 100 people, what is the probability that their mean height is greater than 65 inches? We happen to know the population mean is 64 inches and population standard deviation is 4 inches.

3. Calculate the probability from a Normal distribution: $P(\bar{X} \geq 65)$



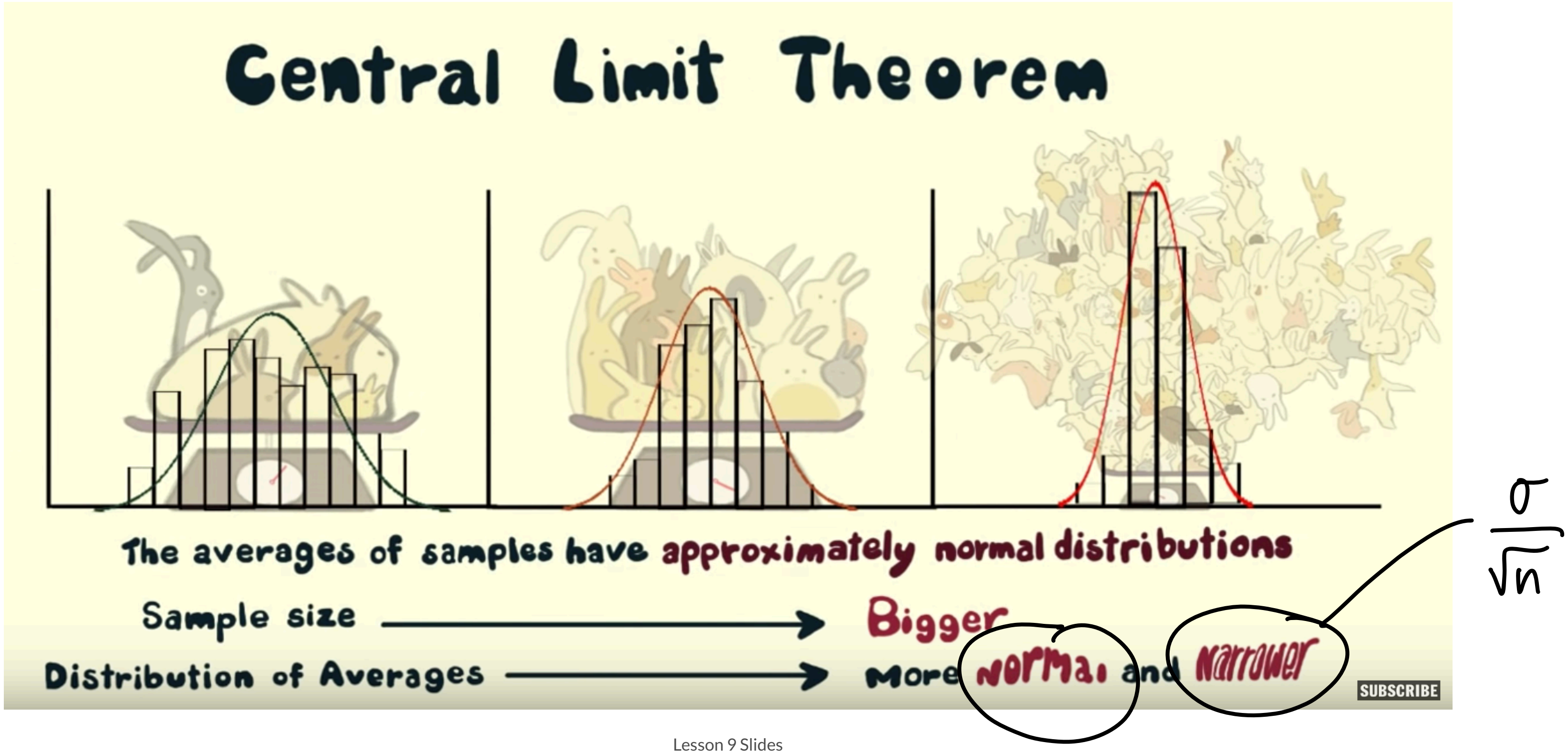

```
pnorm(q = 65, mean = 64, sd = 0.4,
      lower.tail = F)
```

```
[1] 0.006209665
```

The probability that a 100-person sample has a mean of 65 or greater is 0.006. Makes me question if our sample really came from the population…

# Check out this video explanation of CLT

- Bunnies, Dragons and the 'Normal' World: Central Limit Theorem
  - Creature Cast from the New York Times
  - https://www.youtube.com/watch?v=jvoxEYmQHNM&feature=youtu.be



# Summary Review: Point Estimate Terminology

- **Population mean:** $\mu$
- **Population standard deviation:** $\sigma$
- **Sample mean:** $\overline{x}$
- **Sample standard deviation:** $s$
- **Sampling distribution**: Distribution of sample means for repeated samples.
  - Use $\overline{X}$ as the RV for this distribution
  - $\overline{X} \sim \text{Normal}\left(\mu_{\overline{X}} = \mu, \sigma_{\overline{X}} = SE = \frac{s}{\sqrt{n}}\right)$
- **Standard error (SE)**: The standard deviation of the sampling distribution.
  - Formula: $SE = \frac{s}{\sqrt{n}}$