Homework 8

BSTA 550

2023-12-07

> This homework is optional!!
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> This homework is completely optional!! These chapters will be on the test! I cannot stress enough how important these distributions and the CLT are!!

Directions

Please turn in this homework on Sakai. Please submit your homework in pdf format. You can type your work on your computer or submit a photo of your written work or any other method that can be turned into a pdf. Please let me know if you greatly prefer to submit a physical copy. We can work out another way for you to turn in homework.

Try to complete all of the problems listed below at some point this quarter! You may want to save some of them for studying later! Only turn in the ones listed in the "Turn In" column. Please submit problems in the order they are listed.

The more work you include that shows your thought process, the more I can give you feedback.

Chapter	Turn In	Extra Problems
31	TB # 18	TB # 13, 14, 17
32	TB # 8	TB # 3, 5, 10[1], 15
33	NTB # 1	TB # 3, 9, 10
35	TB # 10, NTB # 2	TB # 6, 9, 24
36	TB # 12[2], 14	TB # 4, 11, 13, 15, 16
37	TB # 24, 30	TB # 2, 4, 13, 20, 29

[1] Assume X and Y are independent.
[2] Assume the distances between the cars are independent.

* Although within Chapter 26, these exercises are primarily practicing the material from Chapter 25.

** For Ch 27 # 12, in order to find the conditional densities in parts (a) and (b), you will need to calculate $f_Y(y)$ for the specific regions of y specified. After finding the conditional densities in parts (a) and (b), also calculate the conditional probabilities below. Please submit these together with your other work in parts (a) and (b): Find $\mathbb{P}[0.5 < X < 3|Y = 4]$. Find $\mathbb{P}[0.5 < X < 3|Y = 7]$.

Non-textbook problems (NTB)

1. Suppose that voters arrive at a polling station at the rate of 120 per hour.For each of the following parts, give the name and parameter(s) of the distribution to be used to model the event and set up the expression to find the specified probability. *You do not need to compute the probability.*

   a. The probability that the next voter will arrive in less than 30 seconds.

   b. The probability that 200 voters will arrive within two hours of each other.

   c. The probability that the 50^{th} voter will arrive in between 15 and 30 minutes.

2. The automatic opening device of a military cargo parachute has been designed to open when the parachute is 200 m above the ground. Suppose opening altitude actually has a normal distribution with mean value 200 m and standard deviation 30 m. Equipment damage will occur if the parachute opens at an altitude of less than 100 m. What is the probability that there is equipment damage to the payload of at least one of the five independently dropped parachutes?

Some select answers

Selected answers (or hints) not provided at the end the book:

- Chapter 25
  - # 4: 7/16
  - # 8: (a) $\frac{25}{228}$ (b) $f_X(x) = \frac{1}{12}(x+1)$, for $0 \le x \le 4$ (c) $f_Y(y) = \frac{3}{76}(y^2+1)$, for $0 \le y \le 4$
  - # 18: 5/6
  - # 24: (a) $f_X(x) = -2e^{-2x} + 2e^{-x}$, for $x \ge 0$ (b) $f_Y(y) = 2e^{-2y}$, for $y \ge 0$
- Chapter 26

- # 12: (b) $\frac{233}{256}$ (c) $\frac{65}{256}$ (d) $\frac{1}{512}$
- # 20: (a) Yes. (b) $\frac{15}{16}$
- NTB # 3: (b) 0.09999546 (d) $f_Z(z) = \left(\frac{11}{5} - \frac{2z}{5}\right)e^{-2z}$, for what values of z?

- Chapter 27
  - # 6: $f_{X|Y}(x|y) = \frac{e^{-x/4-y/5}}{4(e^{-y/5}-e^{-9y/20})}$, for $0 < x < y$
  - # 8: $f_{X|Y}(x|y) = \frac{1-x^2}{1-y-\frac{(1-y)^3}{3}}$, for $0 \leq x, 0 \leq y, x + y \leq 1$
  - # 12: (a) $f_{X|Y}(x|y) = \frac{1}{2}$ (c) $\frac{4}{7}$

- Chapter 28
  - # 10: (a) 8/9 (b) 14/3
  - # 18: 4/5

- Chapter 29
  - # 10: (a) 26/81 (b) 74/9
  - # 14: (a) 67/3 (b) 1/14 (c) 25/12 (d) $\sqrt{25/12}$
  - # 26: 250
  - # 32: See notes (or book) for the proof from the discrete random variables case. The proof doesn't depend on what type of random variable (discrete vs. continuous) is being used.
  - NTB # 3: (a) 63 (b) 287/3 (c) -1, 41/3 (d) -7, 287/3

- Chapter 30
  - # 4: $f_x(x) = 1/2$ for $2 \leq x \leq 4$
  - # 8: (a) T (b) T (c) F
  - # 10: (a) F (b) T
  - # 12: (a) T (b) T (c) F (d) T

- Chapter 31
  - # 14: (a) 0.25 (b) 0.02887 (c) 0.063 (d) 0.0145 (e) 0.01625 (f) 0.0055 (f) 6.195 (g) 0.00433 (h) 61.95 (i) 0.0433
  - # 17: 2.25

- # 18: 7/15

- Chapter 32
  - # 8: 0.2526
  - # 5: 0.8047
  - # 10: 0.4323

- Chapter 33
  - #10: (a) $f_x(x) = \frac{x}{9}e^{-x/3}$ for $x > 0$ (b) 0.4963

- Chapter 35
  - # 6: (a) 0 (b) -1.13 (c) ± 0.32
  - # 10: (a) 0.0475 (b) 0.0475 (c) 0.2283 (d) 68.97 to 81.03 (e) 48 to 102 (f) 68.97
  - # 24: (a) 0.2119 (b) 0.0011
  - NTB # 5: 0.002

- Chapter 36
  - # 4: 0.0044
  - # 12: (a) 0.9525 (b) 0.7939 (c) 0.7939
  - # 14: 0.5911
  - # 16: (a) $R = 8.225\sigma + 25\mu$ (b) $R = 16.45\sigma + 100\mu$ (c) $R = 164.5\sigma + 10,000\mu$ (d) $R = 1.645\sqrt{n}\sigma + n\mu$

- Chapter 37
  - # 2: 0.8869
  - # 4: 0.0023
  - # 20: 0.3936
  - # 24: 0.4562
  - # 30: (b) 0.0022 (c) $478.696 \approx 479$