

Combinatorics Practice

Chapter 22

The following problems are for you to practice solving more combinatorics problems. The * indicates problems that are more advanced.

- How many integers between 100 and 999 (inclusive)
 - have distinct digits?
 - have distinct digits and are odd numbers?
- An octave contains 12 distinct notes (on a piano, five black keys and seven white keys).
 - How many different eight-note melodies within a single octave can be written using the white keys only?
 - How many different eight-note melodies within a single octave can be written if the black keys and white keys need to alternate?
- Suppose that a seemingly interminable German opera is recorded on all six sides of a three-record album. In how many ways can the six sides be played so that at least one is out of order?
- The crew of *Apollo 17* consisted of two pilots and one geologist. Suppose that NASA had actually trained a total of nine pilots and four geologists. How many possible *Apollo 17* crews could have been formed?
 - Assume that the pilot positions have identical duties.
 - Assume that the pilot positions are really a pilot and a copilot.
- Ten basketball players meet in the school gym for a pickup game. How many ways can they form two teams of five each?
- A bridge hand (13 cards) is dealt from a standard 52-card deck. Let A be the event that the hand contains four aces; let B be the event that the hand contains four kings. Find $\mathbb{P}(A \cup B)$.
- A committee of 50 politicians to be chosen from among our 100 U.S. Senators. If the selection is done at random, what is the probability that each state will be represented?
- * Suppose that 10 people, including you and a friend, line up for a group picture. How many ways can the photographer rearrange the line if she wants to keep exactly three people between you and your friend?
- * In how many ways can the letters in MISSISSIPPI be arranged so that no two I's are adjacent?
- * If the letters in the familiar phrase STATISTICS IS FUN are arranged at random, what are the chances that not all the S's will be adjacent? (ignore spaces)