Chapter 7: Discrete vs. Continuous Random Variables

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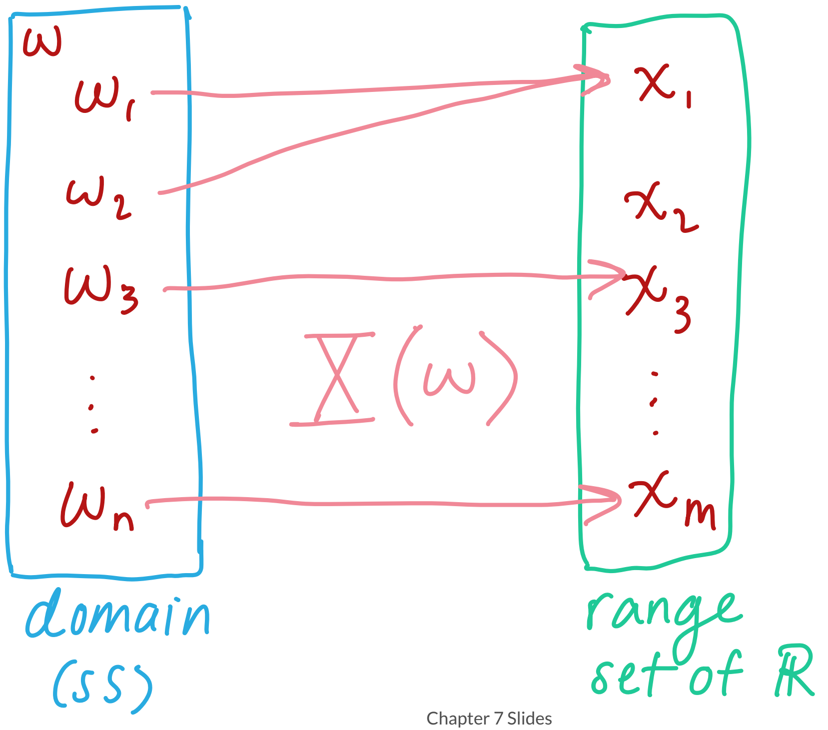
Learning Objectives

1. Map the sample space to the set of real numbers using a discrete and continuous random variable
2. Distinguish between discrete and continuous random variables from a written description

What is a random variable?

Definition: Random Variable

For a given sample space S, a **random variable** (r.v.) is a function whose domain is S and whose range is the set of real numbers $\mathbb{R}$. A random variable assigns a real number to each outcome in the sample space.



X = capital X

RV function

$$\underbrace{X}_{\text{random part}}(\omega) = \underbrace{x}_{\text{outcome of RV}}$$

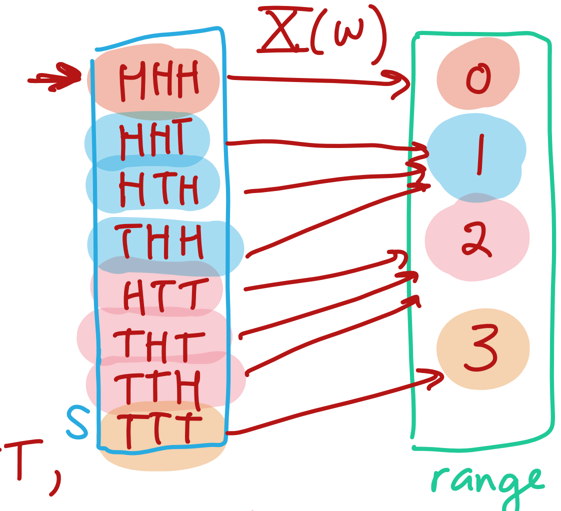
Let's demonstrate this definition with our coin toss

Example 1

Suppose we toss 3 fair coins.

1. What is the sample space?
2. What are the probabilities for each of the elements in the sample space? *Order matters for now*
3. What are the probabilities that you get 0, 1, 2, or 3 tails?

③ Let X = # of tails appear



$X(\omega) = a + b + c$

0 : H

1 : T

$P(X=0) = P(HHH) = 1/8$

$P(X=1) = P(HHT) + P(HTH) + P(THH) = \frac{1}{8} + \frac{1}{8} + \frac{1}{8} = 3/8$

$P(X=2) = P(HTT) + P(THT) + P(TTH) = 3/8$

$P(X=3) = P(TTT) = \frac{1}{8}$

① sample space:

$S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$

② $P(HHH) = \frac{|HHH|}{|S|} = \frac{1}{8}$

$\left(\frac{1}{2}\right) \cdot \left(\frac{1}{2}\right) \cdot \left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^3 = \frac{1}{8}$

Let's stretch our definition of random variables

Example 2

What are some other random variables we could consider in Example 1?

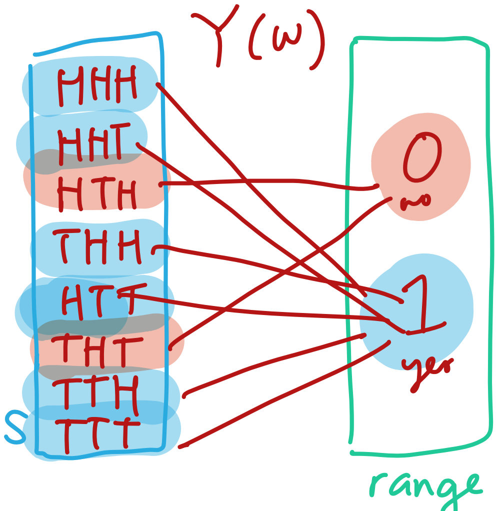
$Y(\omega) = \underline{y}$ that we get two in a row of same face

yes or no

yes → 1

no → 0

$P(Y=1) = \frac{3}{4}$



$Y(\omega) = \begin{cases} 1 & \text{get at least two in a row} \\ 0 & \text{otherwise} \end{cases}$

Some remarks on random variables

- A random variable's value is completely determined by the outcome ω, where $\omega \in S$
  - What is *random* is the outcome ω
- A random variable is a function from the sample space (with outcomes ω) to the set of real numbers
  - We typically write X instead of $X(\omega)$, where X is our random variable
- For example, if we roll three dice, there are $6^3 = 216$ possible outcomes (which is ω)
  - We can define a random variable as the sum of the of the three dice
  - If our outcome is the set of numbers the dice landed on ($\omega = (a, b, c)$), then

$$X(\omega) = X = a + b + c$$

d1 d2 d3

Let's look at a continuous R.V.

Example 3

Let X = how many hours you slept last night.

1. What is the sample space S?
2. What is the range of possible values for X?
3. What is $X(\omega)$?

X = hours slept

ω = hours slept ("random" component)

1. $S = \{\underline{\omega \geq 0}\}$

2. $X(\omega) = 1 \cdot \omega$ $\qquad X \in [0, \infty)$

in $\qquad$ includes 0 (from ≥ 0)

3. $X(\omega) \geq 0$ $\qquad \omega \geq 0$

$X \geq 0$

Discrete vs. Continuous r.v.'s

- For a **discrete** r.v., the set of possible values is either finite or can be put into a countably infinite list
  - You could *theoretically* list the specific possible outcomes that the variable can take
  - If you sum the rolls of three dice, you must get a whole number. For example, you can't get any number between 3 and 4.

- **Continuous** r.v.'s take on values from continuous *intervals*, or unions of continuous intervals
  - Variable takes on a range of values, but there are infinitely possible values within the range
  - If you keep track of the time you sleep, you can sleep for 8 hours or 7.9 hours or 7.99 hours or 7.999 hours …

$P(7.0 \leq X \leq 7.01)$