Chapter 35: Normal Random Variables

Meike Niederhausen and Nicky Wakim

2023-11-27

Table of contents

Learning Objectives

1. Translate a word problem into probability within Normal RV
2. Calculate probabilities within Normal RV using R

Properties of Normal RVs

In cont, $P(X=x) \neq f_X(x)$

- No scenario description here because the Normal distribution is so universal
  - Central Limit Theorem (next class) makes it applicable to many types of events
- Shorthand: $X \sim \text{Normal}(\mu, \sigma^2)$

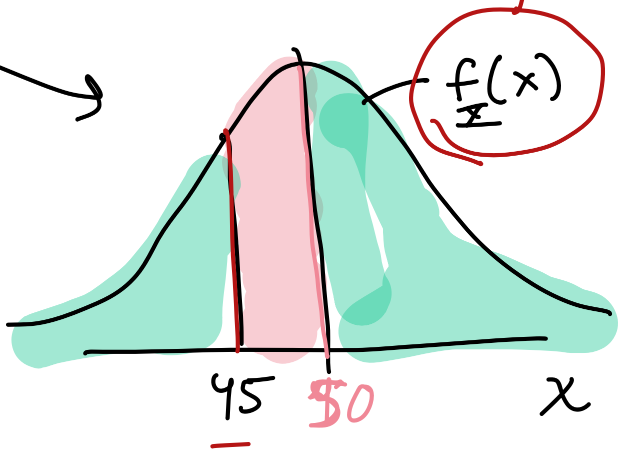
Normal(μ, σ) → stand dev.

variance

$$f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(x-\mu)^2/(2\sigma^2)}, \text{ for } -\text{inf} < x < \text{inf}$$

$$E(X) = \mu$$

$$\text{Var}(X) = \sigma^2$$



Helpful R code

Let's say we're measuring the high temperature today. The average high temperature on this day across many, many years is 50 degrees with a standard deviation of 4 degrees.

- If we want to know the probability that the high temperature is below 45 degrees: $P(X \le 45)$

```
pnorm(q = 45, mean = 50, sd = 4)
```

```
[1] 0.1056498
```

- If we want to know the temoerature, say t, where the probability of that the temperature is at t or lower is 0.35: $P(X \le t) = 0.35$

```
qnorm(p = 0.35, mean = 50, sd = 4)
```

```
[1] 48.45872
```

- If we want to know the probability that the temperature is between 45 and 50 degrees: $P(45 \le X \le 50)$

```
pnorm(q = 50, mean = 50, sd = 4) - pnorm(q = 45, mean = 50, sd = 4)
```

```
[1] 0.3943502
```

- If we want to sample 20 days' temperature (over the years) from the distribution:

```
rnorm(n = 20, mean = 50, sd = 4)
```

```
 [1] 50.69640 52.42826 50.18311 52.45207 52.46715 60.30689 48.92252 53.97830
 [9] 48.51508 49.10167 51.90440 55.46195 50.51701 54.09617 43.67940 52.47262
[17] 50.48654 55.12716 51.37001 53.71046
```

Movie night while studying

Example 1

Children's movies [are normally dist.] run an average of 98 minutes with a standard deviation of 10 minutes. You check out a random movie from the library to entertain your kids so you can study for your test. Assume that your kids will be occupied for the entire length of the movie.

a. What is the probability that your kids will be occupied for at least the 2 hours you would like to study?

b. What is range for the bottom quartile (lowest 25%) of time they will be occupied?

$X \sim N(\mu = 98, \sigma = 10)$

$\hookrightarrow \sigma^2 = 100$

a) $P(X > 2 \cdot 60) = P(X > 120)$

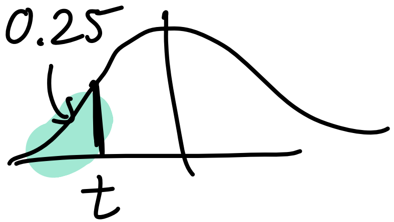
calc in R:

0.0139

b) $P(X \le t) = 0.25$

calc in R qnorm( )

$t = \underline{91.255}$



$(-\infty, 91.255]$

0

Standard Normal Distribution

$$Z \sim \text{Normal}(\mu = 0, \sigma^2 = 1)$$

- Used to be more helpful when computing was not as advanced
  - Use tables of the standard normal
  - You can convert any normal distribution to a standard normal through transformation

$X \sim N(\mu_X, \sigma_X^2) \rightarrow Z = \dfrac{X - \mu_X}{\sigma_X} \sim N(0, 1)$

- $Z = \dfrac{X - \mu_X}{\sigma_X}$
  - Comes from $X = \sigma_X Z + \mu_X$
  - Since σ_X and μ_X are constants, then $E(X) = \mu_X$ and $SD(X) = \sigma_X SD(Z) = \sigma_X$

$$E(X) = \underbrace{\sigma_X E(Z)}_{0} + \underline{\mu_X}$$