Chapter 36: Sums of Independent Normal RVs

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Learning Objectives

1. Calculate probability of a sample mean using a Normally distributed population

Sum of Normal RVs

Theorem 1

Let $X \sim N(\mu, \sigma^2)$, and let $Y = aX + b$, where a and b are constants. Then

$$Y \sim N(a\mu + b, a^2\sigma^2)$$

Theorem 2

Let $X_i \sim N(\mu_i, \sigma_i^2)$ be independent normal rv's, for $i = 1, 2, \ldots, n$. Then

$$\sum_{i=1}^{n} X_i \sim N\left(\sum_{i=1}^{n} \mu_i, \sum_{i=1}^{n} \sigma_i^2\right)$$

$Var(\Sigma) = \Sigma\, Var$ (σ_i^2)

$E(\Sigma X_i) = \Sigma\, E(X_i) = \Sigma \mu_i$

Special Cases

1. Let $X_i \sim N(\mu, \sigma^2)$ be iid normal rv's, for $i = 1, 2, \ldots, n$. Then

$$\sum_{i=1}^{n} X_i \sim N\left(n\mu, n\sigma^2\right)$$

$\sum_{i=1}^{n} \mu = n\mu \qquad \sum_{i=1}^{n} \mu_i = \sum_{i=1}^{n} \mu_i$

2. Let $X_i \sim N(\mu, \sigma^2)$ be iid normal rv's, for $i = 1, 2, \ldots, n$. Then

$$\bar{X} = \frac{\sum_{i=1}^{n} X_i}{n} \sim N\left(\mu, \sigma^2/n\right)$$

showed this in general in HW 6?

3. Let $X \sim N(\mu_X, \sigma_X^2)$, and $Y \sim N(\mu_Y, \sigma_Y^2)$. Then $X \perp Y$

$$X - Y \sim N\left(\mu_X - \mu_Y, \sigma_X^2 + \sigma_Y^2\right)$$

$Var(X - Y) = Var(X) + Var((-1)Y)$

$(-1)^2$

Detecting and solving sums of Normal RVs from a word problem

Example 1

Glaucoma is an eye disease that is manifested by high intraocular pressure (IOP). The distribution of IOP in the general population is approximately normal with mean 16 mmHg and standard deviation 3 mmHg.

1. Suppose a patient has 40 IOP readings. What is the probability that their average reading is greater than 20.32 mmHg, assuming their eyes are healthy?
2. Repeat the previous question for a patient with 10 IOP readings.