Chapter 24: Continuous r.v.'s and pdf's

Recall from Chapter 7:

Discrete vs. Continuous r.v.'s

- For a **discrete** r.v., the set of possible values is either finite or can be put into a countably infinite list.
- **Continuous** r.v.'s take on values from continuous *intervals*, or unions of continuous intervals.

How to define probabilities for continuous r.v.'s?

Definition 24.1 (Probability density function).
*The probability distribution, or **probability density function (pdf)**, of a continuous random variable X is a function $f_X(x)$, such that for all real values a, b with $a \leq b$,*

$$\mathbb{P}(a \leq X \leq b) = \int_a^b f_X(x)dx$$

Remarks:

(1) Note that $f_X(x) \neq \mathbb{P}(X = x)$!!!

(2) In order for $f_X(x)$ to be a pdf, it needs to satisfy the properties
- $f_X(x) \geq 0$ for all x
- $\int_{-\infty}^{\infty} f_X(x)dx = 1$

Example 24.2. *Let $f_X(x) = 2$, for $a \leq x \leq 3$.*

(1) *Find the value of a so that $f_X(x)$ is a pdf.*

(2) *Find $\mathbb{P}(2.7 \leq X \leq 2.9)$.*

(3) *Find $\mathbb{P}(2.7 < X \leq 2.9)$.*

(4) *Find $\mathbb{P}(X = 2.9)$.*

(5) *Find $\mathbb{P}(X \leq 2.8)$.*

Definition 24.3 (Cumulative distribution function).
*The **cumulative distribution function (cdf)** of a continuous random variable X, is the function $F_X(x)$, such that for all real values of x,*

$$F_X(x) = \mathbb{P}(X \leq x) = \int_{-\infty}^{x} f_X(s)ds$$

Example 24.4. *Let $f_X(x) = 2$, for $2.5 \leq x \leq 3$. Find $F_X(x)$.*

Remarks:
In general, $F_X(x)$ is increasing and

- $\lim_{x\to-\infty} F_X(x) = 0$
- $\lim_{x\to\infty} F_X(x) = 1$

Theorem 24.5.
If X is a continuous random variable with pdf $f_X(x)$ and cdf $F_X(x)$, then for all real values of x at which $F'_X(x)$ exists,

$$\frac{d}{dx}F_X(x) = F'_X(x) = f_X(x)$$

Example 24.6. *Let X be a r.v. with cdf*

$$F_X(x) = \begin{cases} 0 & x < 2.5 \\ 2x - 5 & 2.5 \leq x \leq 3 \\ 1 & x > 3 \end{cases}$$

Find the pdf $f_X(x)$.

Solution:

Example 24.7. *Let X be a r.v. with pdf $f_X(x) = 2e^{-2x}$, for $x > 0$.*

(1) *Show $f_X(x)$ is a pdf.*

(2) *Find $\mathbb{P}(1 \leq X \leq 3)$.*

(3) *Find* $F_X(x)$.

(4) *Given* $F_X(x)$, *find* $f_X(x)$.

(5) *Find* $\mathbb{P}(X \geq 1 | X \leq 3)$.

(6) *Find the median of the distribution of* X.