1. Three couples and four single people are at a party. If the people present are seated in a row, how many arrangements are there if
   (a) each coupled person sits next to their partner.
   (b) all the single people sit next to each other, but not necessarily the couples.

2. In a T-maze, a laboratory rat is given the choice of going to the left and getting food or going to the right and receiving a mild electric shock. Assume that before any conditioning (in trial number 1) rats are equally likely to go the left or to the right. After having received food on a particular trial, the probability of going to the left and right become 0.6 and 0.4, respectively on the following trial. However, after receiving a shock on a particular trial, the probabilities of going to the left and right on the next trial are 0.8 and 0.2, respectively. What is the probability that the animal will turn left on trial number 2?

3. Suppose there is a 20% chance that OHSU patients have already had their flu shots, independently of each other. An OHSU clinician is seeing 2 patients today. Let X be the number of her two patients that have already had their flu shots. Now suppose a third patient is added to her schedule, and let Z be the number of her three patients that have already had their flu shots (the three include the first two).
   (a) Make a table of the joint pmf of X (rows) and Z (columns).
   (b) Find $p_{X|Z}(x = 1, z = 1)$.
   (c) Find $p_{X|Z}(x = 1, z = 2)$.
   (d) Find $F_{X,Z}(x = 1, z = 2)$.
   (e) In one sentence, explain in context of the problem what $F_{X,Z}(x = 1, z = 2)$ is the probability of.
   (f) Are X and Z independent? Why or why not?

4. Political campaign workers for the Portland mayoral race are given a list of 3,000 phone numbers of people to call to ask for money. An anonymous donor has already given $5,000 under the condition that the 3,000 people are called. Suppose each of the 3,000 people will donate money independently of the others, and that there is a 50% chance they will donate $20, 25% chance they will donate $100 and a 25% chance they will donate nothing.

   You must define every random variable you use and prove the mean and variance for each random variable.

   (a) Find the expected value of the total amount of money they will raise by making the phone calls.
   (b) Find the variance of the total amount of money they will raise by making the phone calls.

5. Let $X_i \sim$ Negative Binomial(r_i, p) be independent r.v.'s for $i = 1, \ldots, m$. Let $X = \sum_{i=1}^{m} X_i$.
   (a) What does the r.v. X count (model)?
   (b) What is the distribution of X? Make sure to specify the parameter(s) of X's distribution.

   Make sure to show your work for (b) and (c). However, you may use without proof what you know about the mean and variance of each X_i.

   (c) Find $\mathbb{E}[X]$.
   (d) Find $Var[X]$.

6. Two pumps connected in parallel fail independently of one another on any given day. The probability that *only* the older pump will fail is 0.10, and the probability that *only* the newer pump will fail is 0.05. What is the probability that the pumping system will fail on any given day (which happens only if *both* pumps fail)?
   Hint: Draw a Venn diagram.

1. If n girls and n boys randomly sit in a circle of chairs,

   (a) what is the probability that none of the girls are adjacent and none of the boys are adjacent?

   (b) what is the probability that all the girls sit together and all the boys sit together?

2. Smartphones are shipped to a wholesale supplier in batches of 1,000. Suppose that 5% of batches contain one defective phone, 3% contain two defective phones, and 1% contain three defective phones. Five phones from a batch of 1,000 are randomly selected and tested. What is the probability that there are 3 defective phones in the batch given that two of the five tested phones are defective?

   Set up the expression to find the probability. Do not compute it.

3. A researcher is studying a rare disease that occurs in 0.1% of the population independently of each other. Suppose the researcher tests 9,000 study participants to see whether they have the disease or not.

   (a) How many people should the researcher expect to have the disease?

   (b) Set up an expression for the *exact* probability that more than 12 people will have the disease.

   (c) Set up an expression for the *approximate* probability that between 8 and 12 people (inclusive) will have the disease.

4. A study on long term opioid therapy is recruiting Veterans stratified by urban and rural clinics. There are various eligibility criteria, and based on past data we expect 20% of Veterans at urban clinics to be eligible and 30% of Veterans at rural clinics to be eligible, all independently of each other.
   Let U be the number of urban clinic Veterans screened until the next eligible participant is found, and R be the number of rural clinic Veterans screened until the next eligible participant is found.

   (a) Set up an expression for the probability that at least 4 urban clinic Veterans need to be screened until an eligible participant is found?

   (b) Set up an expression for the probability that between 10 and 20 urban clinic Veterans need to be screened until the third urban participant is found?

   (c) Set up an expression for the probability that fewer than 7 urban clinic Veterans and fewer than 5 rural clinic Veterans need to be screened until an eligible urban clinic and an eligible rural clinic participant is found.

   (d) Find the conditional pmf of R given U.

   (e) Find the cdf of U.

   (f) Find the joint cdf of U and R.

   (g) In one sentence, explain in words what $F_{U,R}(5,2)$ is the probability of.

5. A researcher is creating a budget for a study. The fixed costs to cover the salaries of the research team and other expenses will be $250,000. In addition, each study participant will get a $20 gift card for their first study visit and a $30 gift card for their second study visit. 1,000 participants will be enrolled in the study, of which all will complete the first visit. However, there is only an 80% chance that the participants will complete the second visit, independently of each other.

   You must define every random variable you use and prove the mean and variance for each random variable. You do not need to simplify the arithmetic for your final answers.

   (a) What is the expected cost for the study?

   (b) What is the variance of the cost for the study?

1. The German word for probability theory is

$$W\ A\ H\ R\ S\ C\ H\ E\ I\ N\ L\ I\ C\ H\ K\ E\ I\ T\ S\ T\ H\ E\ O\ R\ I\ E$$

   If the letters in this word are arranged at random,

   (a) what is the probability that none of the H's will be adjacent?

   (b) what is the probability that not all of the H's will be adjacent?

2. A chemist is interested in determining whether a certain trace impurity is present in a product. An experiment has a probability of 0.80 of detecting the impurity if it is present. The probability of not detecting the impurity if it is absent in 0.90. The probabilities of the impurity being present and being absent are 0.40 and 0.60 respectively. Five separate experiments result in only three detections. What is the probability that the impurity is present? You only need to set up the expression to find the probability, you do not need to compute it.

3. Deep in the depths of a student's backpack is a collection of pens in different colors. There are 5 black, 4 blue, 3 green, and 2 purple pens.

   (a) If the student randomly selects 8 pens from the backpack without replacement, what is the probability that they took 2 black, 3 blue, 2 green, and 1 purple pens?

   (b) If the student randomly selects 8 pens from the backpack with replacement (returning the selected pen after each draw), what is the probability that they took a purple pen 3 times?

   (c) If the student randomly selects pens from the backpack with replacement (returning the selected pen after each draw) until they get the 3^{rd} green pen, what is the probability that they get the 3^{rd} green pen on the 10^{th} draw?

   (d) Let the random variable X denote how many random draws from the backpack with replacement (returning the selected pen after each draw) are needed until the first green pen is selected. Derive the expected value of the r.v. X.

4. Consider some special 3-sided dice. Roll two of these dice. Let X denote the minimum of the two values that appear, and let Y denote the maximum of the two values that appear.

   (a) Set up the table of values for the joint pmf of X and Y.

   (b) Set up the table of values for the joint cdf of X and Y.

   (c) In one sentence, explain in words what $F_{X,Y}(2,3)$ is the probability of.

   (d) Set up the table of values for the conditional pmf of X given Y.

   (e) In one sentence, explain in words what $p_{X|Y}(2|3)$ is the probability of.

   (f) Find the expected value of Y.

   (g) Find the variance of Y.

5. A tour group is planning a visit to the city of Landport and needs to book 30 hotel rooms. The average price of a room is \$200 with standard deviation \$10. In addition there is a 10% tourism tax for each room.

   You must define every random variable you use. You do not need to simplify the arithmetic for your final answers.

   (a) What is their expected cost for the 30 hotel rooms?

   (b) What is the variance of the cost for the 30 hotel rooms?

1. The director of a choral group decided that at every rehearsal the choir would sit in a different configuration. There are four sections in the choir: basses (B), tenors (T), altos (A), and sopranos (S). The seating at rehearsal has the director in the middle with 2 rows of singers on both sides of him facing towards the center. The diagram below is an example with 4 singers in each of the sections.

B	T	director	A	S
B	T		A	S
B	T		A	S
B	T		A	S

(a) Assuming there is assigned seating within each section, how many ways are there to arrange the sections for rehearsal?

(b) A problem that arose with this seating plan was that one couldn't hear the women singing if men were sitting in front of them, and it was decided that no seating plans with women sitting behind men would be allowed. For example, the following arrangement would not be allowed:

A	T	director	B	S
A	T		B	S

...

With this restriction, how many ways are there to arrange the sections for rehearsal (assuming there is assigned seating within each section)?

For the remaining parts, assume men can sit in front of women (as in part (a))

(c) The choir consists of 10 basses, 9 tenors, 11 altos, and 12 sopranos. How many seating arrangements are possible if singers have to sit together within their respective sections but do not have assigned seating?

(d) To complicate matters, there are actually two parts within each section. There are 5 B1's, 5 B2's, 4 T1's, 5 T2's, 5 A1's, 6 A2's, 6 S1's, and 6 S2's. How many ways are there to arrange the singers if they are sitting not only within their sections, but the singers on the 1^{st} part are all together and the singers on the 2^{nd} part are all together?

(e) How many ways are there to arrange the singers if they are sitting within their sections, but members of the same part cannot sit next to each other? For example, a B1 must sit next to a B2 and cannot sit next to B1. Use the numbers on each part given in (d).

2. Two distinguishable dice are thrown n times in succession.

(a) Compute the probability that double 6 appears at least once.

(b) How large need n be to make this probability at least $\frac{1}{2}$?
You do not need to simplify your final answer.

3. A computer in Portland sends a signal to a computer in Dnepropetrovsk. If the computer in Dnepropetrovsk receives the signal, it will send a signal back to the computer in Portland. Assume that one signal in 1,000,000 is lost in transition. If the computer in Portland does not receive an answer, find the probability that the computer in Dnepropetrovsk received the original signal.
For the following, you only need to set up the expressions to find the probability. Do not compute it.

4. The number of people arriving for treatment at an emergency room can be modeled by a Poisson process with an average of five per hour. Suppose that with probability 0.04, an arriving patient will need emergency surgery. Assume that patients arrive independently of each other, and whether they need surgery is independent from person to person and also of the number of people that arrive.
For the following, you only need to set up the expressions to find the probabilities. Do not compute them.
Make sure to clearly define every random variable you use.

(a) What is the probability that at least seven arrivals occur during a particular 2 hour time period?

(b) What is the probability that exactly six arrivals occur during an hour and that none of them need emergency surgery?

(c) For any fixed $x \geq 2$, what is the probability that x arrivals occur during an hour, of which two need emergency surgery?

5. A casino patron will continue to make \$5 bets on red in roulette until she has won 4 of the bets.

   Note: This means that on each bet she will either win \$5 with probability $\frac{18}{38}$ *or lose \$5 with probability* $\frac{20}{38}$.

   Make sure to clearly define every random variable you use. You do not need to simplify your final answers.

   (a) What is the probability that she places a total of 9 bets?

   (b) What is her expected winnings when she stops?
   Make sure to clearly define the random variable for her winnings.

   (c) Find the variance of her expected winnings when she stops.

6. Let $Y = aX + b$ for some random variable X.

   (a) There is a simple formula for the variance of Y in terms of the random variable X. What is this formula?

   (b) Derive the formula in (a) for the variance of Y in terms of the random variable X.

1. Three married couples and four single people are at a party. If the people present are seated in a row, how many arrangements are there if

You must explain your solutions in order to receive full credit. You do not need to simplify your answers.

(a) each married person sits next to their spouse.

(b) all the single people sit next to each other, but not necessarily the couples.

2. Records show that 700 students recently entered an Oregon school district. Of those 700 students, 100 have not received their vaccinations. The district's physician is scheduled to go from school to school next Tuesday to give shots to those who need them. If we know that approximately 3% of the district's students are absent on any given day, what is the probability that more than 10 of the unvaccinated students will miss the doctor's visit?

(a) *Set up* the expression to calculate the exact probability.

(b) *Set up* the expression to approximate the probability.

3. A new drug is packaged to contain 30 pills in a bottle. Suppose that 98% of all bottles contain no defective pills, 1.5% contain one defective pill, and 0.5% contain two defective pills. Two pills from a bottle are randomly selected and tested. What is the probability that there are 2 defective pills in the bottle given that one of the two tested pills is defective?

Set up the expression to find the probability. Do not compute it.

4. Chris tries to throw a ball of paper in the wastebasket behind his back (without looking). He estimates that his chance of success each time, regardless of the outcome of the other attempts, is 1/3. Let X be the number of attempts required. If he is not successful within the first 4 attempts, then he quits, and he lets X =5 in such a case. Let Y indicate whether he makes the basket successfully within the first two attempts. Thus $Y = 1$ if his first or second attempt is successful, and $Y = 0$ otherwise.

(a) What is the probability mass function of X?

(b) What is the probability mass function of Y?

(c) Find $p_{X|Y}(x = 1|y = 0)$, and in one sentence explain in words what this is the probability of.

(d) Find $p_{X|Y}(x = 5|y = 0)$, and in one sentence explain in words what this is the probability of.

(e) Find $p_{X|Y}(x = 2|y = 1)$, and in one sentence explain in words what this is the probability of.

(f) Find $p_{X|Y}(x = 4|y = 1)$, and in one sentence explain in words what this is the probability of.

5. A computer company gets calls for technical assistance on average 10 times per hour during the 9-hour workday and on average 2 times per hour during the remaining 15 hours of the day, independent of the workday calls. Each workday call costs the company \$50 and each evening call costs the company \$100.

Make sure to show all your work and in particular clearly define every random variable you use. Simplify your final answers.

(a) Find the expected cost associated with the calls received during one 24-hour day.

(b) Find the variance of the cost associated with the calls received during one 24-hour day.

6. Let $X_1, X_2, \ldots, X_n$ be independent random variables with common mean μ and variance σ^2. Show that

$$\mathbb{E}\Big[\sum_{i=1}^{n} \frac{(X_i - \mu)^2}{n}\Big] = \sigma^2.$$