Questions from Fall 2008 Exam 2

1. (a) $f_{X,Y}(x,y) = 4e^{-8y}, \quad 2 \le x \le 4, \quad y > 0$
   (b) $\frac{25}{8}$
   (c) $\int_0^\infty \int_2^4 (4x^3y)4e^{-8y}\, dxdy$
2. (a) Erlang($\lambda = 2$,$n = 50$)
$$\mathbb{P}[15 \le X \le 30] = \int_{15}^{30} \frac{(2)^{50}}{49!} x^{49} e^{-2x} dx$$
   Can also answer using a Gamma distribution.
   (b) Exponential($\lambda = \frac{1}{30}$)
$$\mathbb{P}[X < 30] = \int_0^{30} \frac{1}{30} e^{-\frac{1}{30}x} dx$$
   Can also answer using an Erlang or a Gamma distribution.
   (c) Poisson($\lambda = 240$)
$$\mathbb{P}[X = 200] = \frac{240^{200} e^{-240}}{200!}$$
3. 0.0409
4. (a) $M_Y(t) = e^{5\lambda(e^t-1)}$
   (b) Poisson(5λ)
   (c) $M_W(t) = e^{\lambda(e^{5t}-1)}$
   (d) W is not Poisson since its mgf cannot be written in the form of a Poisson mgf, i.e. in the form $e^{\gamma(e^t-1)}$ for some γ.
5.
$$f_{X_{\min}}(x) = \frac{12}{35} e^{-\frac{12}{35}x}, \quad for \quad x > 0$$
6. The only way you have a chance of doing this is by *DRAWING THE PICTURE!!!*

$$\begin{aligned}
\mathbb{P}[|X - Y| \le 0.5] &= \mathbb{P}[X - 0.5 \le Y \le X + 0.5] \\
&= 1 - \int_3^{4.5} \int_{x+0.5}^{5} \frac{3}{784} x^2 y\ dydx - \int_{3.5}^{5} \int_3^{x-0.5} \frac{3}{784} x^2 y\ dydx \\
\underline{\text{OR}} &= 1 - \int_{3.5}^{5} \int_3^{y-0.5} \frac{3}{784} x^2 y\ dxdy - \int_3^{4.5} \int_{y+0.5}^{5} \frac{3}{784} x^2 y\ dxdy \\
\underline{\text{OR}} &= \int_3^{3.5} \int_3^{x+0.5} \frac{3}{784} x^2 y\ dydx + \int_{3.5}^{4.5} \int_{x-0.5}^{x+0.5} \frac{3}{784} x^2 y\ dydx \\
&+ \int_{4.5}^{5} \int_{x-0.5}^{5} \frac{3}{784} x^2 y\ dydx
\end{aligned}$$

7. (F08 Exam 3) $10:52.762$ a.m.

Questions from Fall 2009 Exam 2

1. (a) $f_X(x) = 2x, \quad 0 \leq x \leq 1, \quad f_Y(y) = 3y^2, \quad 0 \leq y \leq 1$

   (b) Yes, since $f_X(x)f_Y(y) = f_{X,Y}(x,y)$.

   (c)
   $$\int_0^1 \int_y^1 (x-y)6xy^2 \; dxdy + \int_0^1 \int_0^y -(x-y)6xy^2 \; dxdy$$
   OR
   $$\int_0^1 \int_0^x (x-y)6xy^2 \; dydx + \int_0^1 \int_x^1 -(x-y)6xy^2 \; dydx$$

2. (a) 0.5572

   (b) 0.9625

   (c) 15.196

3. (a)
   $$f_{X_{\max}}(x) = -0.003e^{-0.003x} + 0.001e^{-0.001x} + 0.002e^{-0.002x}, \quad for \quad x > 0$$

   (b)
   $$\int_0^\infty x(-0.003e^{-0.003x} + 0.001e^{-0.001x} + 0.002e^{-0.002x})dx$$

Questions from Fall 2010 Exam 2

1. (a)

$$F_X(x) = \begin{cases} 0 & x < 0 \\ \frac{x^2}{2} & 0 \le x < 1 \\ 2x - \frac{x^2}{2} - 1 & 1 \le x \le 2 \\ 1 & x > 2 \end{cases}$$

   (b) $\frac{1}{\sqrt{0.4}}$

2. (a) Gamma with $\lambda = 12$ per hour and $r = 10$.

$$\mathbb{P}[X > 3/4] = \int_{3/4}^{\infty} \frac{12^{10}}{9!} x^9 e^{-12x} dx$$

   (b) Exponential with $\lambda = 12$ per hour

$$\mathbb{P}[X < 1/2] = \int_0^{1/2} 12e^{-12x} dx$$

   (c) Poisson with $\lambda = 48$ per 4 hours

$$\mathbb{P}[X \le 30] = \sum_{x=0}^{30} \frac{48^x e^{-48}}{48!}$$

3.

$$1 - \mathbb{P}\left(Z < \frac{39.5 - 1000(.0.1)}{\sqrt{1000(0.99)(0.01)}}\right)$$

4. (a) 49
   (b) 418
   (c) 120

5. (a)

$$\mathbb{P}\left[D > C\right] = \int_0^{20} \int_c^{20} \frac{d}{4000} dddc = \int_0^{20} \int_0^{d} \frac{d}{4000} dcdd$$

   (b) $f_{min}(x) = \frac{1}{20} + \frac{x}{200} - \frac{3x^2}{8000}$ for $0 \le x \le 20$

Questions from Fall 2011 Exam 2 and Final

1. (a)

$$F_X(x) = \begin{cases} 0 & x < \theta \\ 1 - (\frac{\theta}{x})^k & x \geq \theta \end{cases}$$

   (b) $\theta \sqrt[k]{2}$

2. (a) 0.9938
   (b) $\sum_{x=4}^{8} \binom{8}{x} (0.9938)^x (0.0062)^{8-x}$
   (c) 4.10

3. $f_{X_{\min}}(x) = (\sum_{i=1}^{n} \lambda_i) e^{-x \sum_{i=1}^{n} \lambda_i}$, for $x > 0$ (You need to show why this is true instead of citing the result in the book)

4. (a) $\sum_{x=201}^{400} \binom{400}{x} (0.5)^x (0.5)^{400-x}$
   (b) 0.4801

5. (a) Normal($\mu = 2\mu_1 - 3\mu_2 + 5\mu_3$,$\sigma^2 = 4\sigma_1^2 + 9\sigma_2^2 + 25\sigma_3^2$)
   (b) Normal($\mu = \frac{1}{3} \sum_{i=1}^{3} \mu_i$,$\sigma^2 = \frac{1}{9} \sum_{i=1}^{3} \sigma_i^2$)

6. (a) $\mathbb{E}[Min(X_P, X_L)] = \frac{12}{5}$
   (b) $\mathbb{E}[Max(X_P, X_L)] = \frac{115}{12}$

7. (a) $f(x, y) = \frac{1}{2x}$, for $0 < y < x < 2$
   (b) $f_Y(y) = \frac{1}{2}(\ln 2 - \ln y)$, for $0 < y < 2$

Questions from Fall 2012 Exam 2

1.

$$F_X(x) = \begin{cases} 0 & x \leq -2 \\ -\frac{x^2}{16} + \frac{x}{4} + \frac{3}{4} & -2 < x < 2 \\ 1 & x \geq 2 \end{cases}$$

2. (a) 1/4
   (b) 5/4
   (c) 1
   (d) e^{-3}

3. (a) 4.96 to 7.04

(b) 10, 25

(c) 1 ($z = -6$)

4. 0.3472

5. (a) $f_X(x) = 2x$, for $0 < x < 1$

   (b) 2/3

   (c) $\mathbb{E}[2X^2Y] = \int_0^1 \int_0^x 4x^2 y dy dx$ or $\int_0^1 \int_y^1 4x^2 y dx dy$

   (d) $\frac{1}{x}$, for $0 < y < x < 1$

   (e) 0.6

   (f) No. The bounds of $f_{X,Y}(x, y)$ have x dependent on y. Alternatively, you can show that $f_{X,Y}(x, y) \neq f_X(x) f_Y(y)$.

6. (a) $f_{X,Y}(x, y) = \frac{2}{5} e^{-2y}$, for $0 \leq x \leq 5, y > 0$.

   (b) $\mathbb{P}(X < Y) = \int_0^5 \int_x^\infty \frac{2}{5} e^{-2y} dy dx$ or $\int_0^5 \int_0^y \frac{2}{5} e^{-2y} dx dy + \int_5^\infty \int_0^5 \frac{2}{5} e^{-2y} dx dy$ or $1 - \int_0^5 \int_0^x \frac{2}{5} e^{-2y} dy dx$ or $1 - \int_0^5 \int_y^5 \frac{2}{5} e^{-2y} dx dy$

   (c)
$$F_Z(z) = \begin{cases} 0 & z < 0 \\ 1 - \left(1 - \frac{z}{5}\right) e^{-2z} & 0 \leq z \leq 5 \\ 1 & z > 5 \end{cases}$$

Questions from Fall 2014 Exam 2

1.
$$F_X(x) = \begin{cases} 0 & x < 1 \\ \frac{x^2}{2} - x + \frac{1}{2} & 1 < x < 2 \\ 3x - \frac{x^2}{2} - 3.5 & 2 \leq x \leq 3 \\ 1 & x > 3 \end{cases}$$

2. (a) 0.1357

   (b) 59.75

3. (a) $\frac{1}{3}$

   (b) $f_{X,Y}(x, y) = \frac{1}{2x}$ for $0 < y < x < 2$

   (c) $f_Y(y) = \frac{1}{2}(\ln 2 - \ln y)$ for $0 < y < 2$

   (d) $\mathbb{E}[5XY^3] = \int_0^2 \int_0^x 5xy^3 \frac{1}{2x} dy dx$ or $\int_0^2 \int_y^2 5xy^3 \frac{1}{2x} dx dy$

4. (a) $\mathcal{N}(\mu = 2\mu_1 - 3\mu_2 + 5\mu_3, \sigma^2 = 4\sigma_1^2 + 9\sigma_2^2 + 25\sigma_3^2)$
   (b) $\mathcal{N}(\mu = \frac{\mu_1+\mu_2+\mu_3}{3}, \sigma^2 = \frac{\sigma_1^2+\sigma_2^2+\sigma_3^2}{9})$

5. (a) $f_{X,Y}(x,y) = \frac{x}{32}$, for $0 \leq x, y \leq 4$
   (b) $\mathbb{P}(X < Y) = \int_0^4 \int_0^y \frac{x}{32} dxdy$ or $\int_0^4 \int_x^4 \frac{x}{32} dydx$
   (c)

$$F_Z(z) = \begin{cases} 0 & z < 0 \\ 1 - \left(1 - \frac{z^2}{16}\right)\left(1 - \frac{z}{4}\right) & 0 \leq z \leq 4 \\ 1 & z > 4 \end{cases}$$

Questions from Fall 2015 Exam 2

1. 11.68 and 20.32 mm Hg

2. (a) $\mu = 1, \sigma = \sqrt{0.999}$
   (b) $\sum_{x=6}^{1000} \binom{1000}{x} 0.001^x 0.999^{1000-x}$
   (c) $1 - \sum_{x=0}^{5} \frac{e^{-1}1^x}{x!}$

3. (a) 0
   (b) 17
   (c) 6
   (d) 6

4. (a) $\frac{1}{3}$
   (b) $f_{X|Y}(x|y) = \frac{1}{6-3y/2}$ for $2 \leq y \leq 4$ and $0 \leq x \leq \frac{8}{3} - \frac{2}{3}y$
   (c) $\frac{8}{21}$

5. $f_Y(y) = (\sum_{i=1}^n \lambda_i)e^{-y\sum_{i=1}^n \lambda_i}$, for $y > 0$ (You need to show why this is true instead of citing the result in the book)