1. Suppose that the random variables X and Y are independent and that $X \sim U[2,4]$ and $Y \sim \text{Exp}(\lambda = 8)$.

   (a) Find the joint distribution of X and Y.

   (b) Find $\mathbb{E}[3XY + 2]$.

   (c) Set up the equation to find $\mathbb{E}[4X^3Y]$.

2. Suppose that voters arrive at a polling station at the rate of 120 per hour.
   For each of the following parts, give the name of the distribution to be used to model the event and set up the expression to find the specified probability.
   You do not need to compute the probability!!!

   (a) The probability that the 50^{th} voter will arrive in between 15 and 30 minutes.

   (b) The probability that the next voter will arrive in less than 30 seconds.

   (c) The probability that 200 voters will arrive within two hours of each other.

3. A fair coin is tossed 500 times. What is the *approximate* probability that the number of heads appearing is at least 270?
   Use the Normal approximation to the Binomial distribution to answer this question.

4. Let X be a Poisson(λ) random variable with moment generating function

$$M_X(t) = e^{\lambda(e^t - 1)}.$$

   (a) Suppose X_1, X_2, X_3, X_4, and X_5 are i.i.d. random variables with the same distribution as X. Find the moment generating function of $Y = \sum_{i=1}^{5} X_i$.

   (b) Is $Y = \sum_{i=1}^{5} X_i$ also a Poisson random variable? Why or why not? If yes, then what is its parameter?

   (c) Find the moment generating function of $W = 5X$.

   (d) Is $W = 5X$ also a Poisson random variable? Why or why not? If yes, then what is its parameter?

5. A network consists of 2 different jobs that are being completed independently of each other in parallel. The network is completed as soon as one of the jobs is done. Suppose the first job has a completion time that is exponentially distributed with a mean time of 5 hours, and the second job has a completion time that is exponentially distributed with a mean time of 7 hours. Find the probability density function for the time until the first job is completed.

6. Xavier and Yolinda agree to meet at the library between 3:00 and 5:00 pm to study for an upcoming math exam. Let Xavier's arrival time be modeled by the random variable X with pdf $f_X(x) = \frac{3}{98}x^2$ for $3 < x < 5$, and let Yolinda's arrival time be modeled by the random variable Y with pdf $f_Y(y) = \frac{1}{8}y$ for $3 < y < 5$. Suppose their arrival times are independent and that the first one to arrive will wait only half an hour before leaving.
   Set up the equation to find the probability that they will meet each other at the library.

7. (Fall 2008 Exam 3) A professor has three errands to take care of in Waldschmidt Hall. Let X_i = the time that it takes for the i^{th} errand ($i = 1, 2, 3$), and let X_4 = the total time in minutes that she spends walking to and from the building and between each errand. Suppose the X_i ='s are independent, normally distributed, with the following means and standard deviations: $\mu_1 = 15, \sigma_1 = 4, \mu_2 = 5, \sigma_2 = 1, \mu_3 = 8, \sigma_3 = 2, \mu_4 = 12, \sigma_4 = 3$. She plans to leave her office at precisely 10:00 am and wishes to post a note on her office door that reads, "I will return by t a.m." What time t should she write down if she wants the probability of her arriving after t to be 0.01?

Exam #2 — Fall 2009

1. Suppose that the random variables X and Y have joint density

$$f_{X,Y}(x, y) = 6xy^2, \quad 0 \leq x \leq 1,\ 0 \leq y \leq 1.$$

   (a) Find the marginal distributions of X and Y.

   (b) Are X and Y independent? Why or why not?

   (c) Set up the equation to find $\mathbb{E}[\ |X - Y|\ ]$. Simplify the equation to the point where there are no absolute values left in it.

2. A company that manufactures and bottles marion berry juice uses a machine that automatically fills 16-ounce bottles. There is some variation, however, in the amounts of liquid dispensed into the the bottles that are filled. The amount dispensed has been observed to be approximately normally distributed with mean 16 ounces and standard deviation 1.2 ounces.

   (a) What is the probability that a randomly selected bottle will have between 15.5 and 17.5 ounces dispensed into it?

   (b) If 25 bottles are being filled, what is the probability that at least 10 of them will have between 15.5 and 17.5 ounces dispensed into them? (Just set up the expression for this probability; do not calculate it)

   (c) The upper 75% of fill levels contain at least how many ounces in them?

3. Suppose a hallway has two bulbs lighting it, where the lifetimes of the bulbs are independent and can be modeled by Exponential distributions with a mean of 500 hours for the first bulb and a mean of 1000 hours for the second bulb. We want to know how long on average it will take for the hallway to become dark (both bulbs burn out).

   (a) Find the probability density function for the time until both bulbs burn out.

   (b) Set up the equation to find the expected time until both bulbs burn out.

1. (a) Let X be a random variable with probability density function

$$f(x) = \begin{cases} x & 0 \le x < 1 \\ 2 - x & 1 \le x \le 2 \\ 0 & \text{else} \end{cases}$$

   Find the cumulative distribution function of X.

   (b) Let X be a random variable with probability density function $f(x) = \frac{2}{x^3}$, $x > 1$. Find the 60^{th} percentile of X.

2. Suppose that students arrive at the Math Resource Center at the rate of 12 per hour.
For each of the following parts, give the name of the distribution to be used to model the event and set up the expression to find the specified probability.
You do not need to compute the probability!!!

   (a) The probability that the 10^{th} student will arrive in more than 45 minutes.

   (b) The probability that the next student will arrive in less than 30 minutes.

   (c) The probability that at most 30 students will arrive in the next four hours.

3. In response to concerns about nutritional contents of fast foods, McDonald's has announced that it will use a new cooking oil for its french fries that will decrease substantially trans fatty acid levels and increase the amount of more beneficial polyunsaturated fat. The company claims that 99 out of 100 people cannot detect a difference in taste between the new and old oils. Assuming that this figure is correct (as a long-run proportion), what is the *approximate* probability that in a random sample of 1000 individuals who have purchased fries at McDonald's, at least 40 can taste the difference between the two oils?

   Use the Normal approximation to the Binomial distribution to *set up* the probability answering this question. *Your answer should not involve an integral.*

4. Let X and Y be independent random variables, such that

$$X \sim \Gamma(\alpha = 2, \beta = 3), \quad Y \sim \Gamma(\alpha = 4, \beta = 5)$$

   (a) Find the expected value of $X + 2Y + 3$.

   (b) Find the variance of $X + 2Y + 3$.

   (c) Find the expected value of XY.

5. The future lifetime of Cat is uniformly distributed from zero to 20 years, and the future lifetime of Dog can be modeled by the probability density function $f(x) = \frac{x}{200}$, for $0 \le x \le 20$. The two lifetimes are independent.

   (a) Set up the equation for the probability that Dog will outlive Cat.

   (b) Find the probability density function of the smaller of the two lifetimes.

1. A family of pdf's that has been used to approximate the distribution of income, city population size, and size of firms is the Pareto family. The family has two parameters, k and θ, both > 0, and the pdf is

$$f(x) = \begin{cases} \frac{k \cdot \theta^k}{x^{k+1}} & x \geq \theta \\ 0 & x < \theta \end{cases}$$

   (a) Find the cumulative distribution function of a Pareto random variable.

   (b) Find the median of a Pareto random variable.

2. Suppose that scores on a certain manual dexterity test are normal with mean 12 and standard deviation 2.

   (a) What is the probability that a randomly chosen person scores more than 7 on the test? (*Find the probability.*)

   (b) If eight randomly selected individuals take the test, what is the probability that at least half of them score more than 7 on the test? (*Set up the expression for the probability.*)

   (c) What value of c is such that the interval $(12 - c, 12 + c)$ includes 96% of all scores?

3. Let $X_1, X_2, ..., X_n$ be independent exponential random variables, with means $1/\lambda_1, 1/\lambda_2, ..., 1/\lambda_n$ respectively. Find the probability distribution function of $Y = \min(X_1, X_2, ..., X_n)$. You must show all work to receive full credit.

4. (F11 Final) A fair coin is tossed 400 times.

   (a) Set up the expression for the *exact* probability that the number of heads appearing is at least 201?

   (b) Find the *approximate* probability that the number of heads appearing is at least 201? **Use the Normal approximation to the Binomial distribution to answer this question.**

5. (F11 Final) Let $X_1, X_2,$ and X_3 represent the times necessary to perform three successive repair tasks at a certain service facility. Suppose they are independent normal random variables with expected values $\mu_1, \mu_2,$ and μ_3 and variances $\sigma_1^2, \sigma_2^2,$ and σ_3^2, respectively. For the following, state what the distribution of the random variable is.
Make sure to specify the name of the distribution and specify all relevant parameter(s).

   (a) Distribution of $2X_1 - 3X_2 + 5X_3$.

   (b) Distribution of $\bar{X}$.

6. (F11 Final) An insurer finds that the time until occurrence of a claim from its property insurance division is exponentially distributed with a mean of 4 unit of time, and the time until occurrence of a claim from its life insurance division is exponentially distributed with a mean of 6 units of time. Claims occur independently in the two divisions. Find the expected time until the first claim occurrence, property or life.
You must show all of your work to receive full credit.

7. (F11 Final) A stick is 2 feet long. You break it at a point X (measured from the left end) chosen randomly uniformly along its length. Then you break the left part at a point Y chosen randomly uniformly along its length. In other words, X is uniformly distributed between 0 and 2 and, given $X = x$, Y is uniformly distributed between 0 and x.

   (a) Find $f(x, y)$ using $f_X(x)$ and $f_{Y|X}(y|x)$.

   (b) Find $f_Y(y)$.

EXAM #2 — FALL 2012

1. Let
$$f_X(x) = \frac{2-x}{8} \quad \text{for} \quad -2 < x < 2.$$
Find the cumulative distribution function of X.

2. The arrival times of students at the health center at a university is known to be a Poisson distribution with an average rate of 4 sick students arriving per hour.

   (a) What is the expected time that will elapse between the 5th and 6th arrivals?

   (b) What is the expected time that will elapse between the 2nd and 7th arrivals?

   (c) What is the expected number of students who arrive in the next 15 minutes?

   (d) What is the probability that there will be at least 15 minutes between each of the next 4 arrivals? (*Set up the expression for the probability.*)

3. Suppose your waiting time for a bus to go to work in the morning is normally distributed with mean 6 minutes and standard deviation 2 minutes, whereas the waiting time in the evening is normally distributed with mean 4 minutes and standard deviation 1 minute. Assume that waiting times are independent in morning and evening and from day to day.

   (a) What is the range for the middle 40% of morning waiting times?
   You do not need to simplify your final answer.

   (b) What are the expected value and variance of the difference between total morning waiting time and total evening waiting time for a particular work week (5 days)?

(c) What is the probability that for all 10 trips in a work week the average time you wait for the bus is greater than 2 minutes?

4. The number of words that Hector sends to his buddy Tom in text messages is roughly 100 per day. What is the *approximate* probability that, on a given day, he sends between 95 and 105 words to Hector?

5. Teresa and Allison each have arrival times uniformly distributed between 12:00 and 1:00. Their times do not influence each other. If Y is the first of the two times and X is the second, on a scale of 0 to 1, then the joint pdf of X and Y is $f_{X,Y}(x, y) = 2$ for $0 < y < x < 1$.

   (a) Find the marginal density of X.

   (b) Find the expected value of X.

   (c) Set up the expression for $\mathbb{E}[2X^2Y]$.

   (d) Find the conditional density of Y given $X = x$.

   (e) Find the conditional probability that Y is between 0 and 0.3, given that X is 0.5.

   (f) Are X and Y independent? Why or why not?

6. Let X be uniform on $[0, 5]$. Let Y be exponential with mean 0.5. Assume X and Y are independent.

   (a) Find the joint distribution $f_{X,Y}(x, y)$.

   (b) Set up the equation for the probability that X is less than Y.

   (c) Let Z be the random variable that is the smaller of X and Y. Find the cumulative distribution function for Z.
   Make sure to show all your work.

Exam #2 — Fall 2014

1. Let X be a random variable with probability density function

$$f(x) = \begin{cases} x - 1 & 1 \leq x < 2 \\ 3 - x & 2 \leq x \leq 3 \\ 0 & \text{else} \end{cases}$$

   Find the cumulative distribution function of X.

2. At Landport University, a large course has 100 registered students. Each student decides each day (independently of the other students and independent of her/his own prior behavior) whether to attend the class, and the probability a student attends class is only 0.5.

(a) *Approximate* the probability that strictly more than 55 students will attend class on a particular day.

(b) Due to space issues on campus, the registrar wants to schedule a classroom that holds less than 100 students for this class, but the professor insists on having enough seats so the probability is 98% (or more) of no overflow on a given day. How many seats does the classroom need to have?
You do not need to simplify your final answer.

3. A stick is 2 feet long. You break it at a point X (measured from the left end) chosen randomly uniformly along its length. Then you break the left part at a point Y chosen randomly uniformly along its length. In other words, X is uniformly distributed between 0 and 2 and, given $X = x$, Y is uniformly distributed between 0 and x.

   (a) Find the conditional probability that Y is between 1/2 and 1, given that X is 1.5.

   (b) Find $f(x, y)$ using $f_X(x)$ and $f_{Y|X}(y|x)$.

   (c) Find the marginal density of Y.

   (d) Set up the expression for $\mathbb{E}[5XY^3]$.

4. Let X_1, X_2, and X_3 represent the times necessary to perform three successive repair tasks at a certain service facility. Suppose they are independent normal random variables with expected values μ_1, μ_2, and μ_3 and standard deviations σ_1, σ_2, and σ_3, respectively. For the following, state what the distribution of the random variable is.
Make sure to specify the name of the distribution and specify all relevant parameter(s).

   (a) Distribution of $2X_1 - 3X_2 + 5X_3$.

   (b) Distribution of $\bar{X}$.

5. The future lifetime of a cellphone (in years) manufactured by Xenati can be modeled by the probability density function $f_X(x) = \frac{x}{8}$, for $0 \leq x \leq 4$. The future lifetime of a cellphone manufactured by Yeckery is uniformly distributed from zero to 4 years. Assume the lifetimes are independent.

   (a) Find the joint probability density function of the two lifetimes.

   (b) Set up the equation for the probability that a Xenati cellphone's lifetime is less than that of a Yeckery cellphone.

   (c) Find the cumulative distribution function for the minimum of the two lifetimes.
   Make sure to show all your work.

1. Glaucoma is an eye disease that is manifested by high intraocular pressure (IOP). The distribution of IOP in the general population is approximately normal with mean = 16 mm Hg and standard deviation 3 mm Hg. The middle 85% of values are considered to be in the normal range for IOP. Find the cutoff values for the IOP normal range.

   Set up the expressions for the cutoff values. You do not need to calculate the final values.

2. Suppose that the probability of developing a specific type of breast cancer in women aged 40-49 is 0.001. Assume the occurrences of cancer are independent. You have a random sample of 1000 women aged 40-49.

   (a) How many of the 1000 women would you expect to develop this type of breast cancer, and what is the standard deviation? *Set up expressions only.*

   (b) Set up the expression for the *exact* probability that more than 5 of the 1000 women will develop this type of breast cancer.

   (c) Set up the expression for the *approximate* probability that more than 5 of the 1000 women will develop this type of breast cancer.

3. Let X and Y be independent random variables, such that

$$X \sim \Gamma(n = 2, \lambda = 1), \quad Y \sim \Gamma(n = 9, \lambda = 3)$$

   (a) Find the expected value of $2X - 3Y + 5$.

   (b) Find the variance of $2X - 3Y + 5$.

   (c) Find the expected value of XY.

   (d) Find the expected value of X^2.

4. Consider a pair of random variables X,Y with constant joint density on the quadrilateral with vertices (0,0),(3,0),(3,2),(0,4).

   Calculate final answers for all parts of this problem - meaning, calculate all integrals.

   (a) For $0 \leq y \leq 2$, find the conditional density $f_{X|Y}(x|y)$ of X, given $Y = y$.

   (b) For $2 \leq y \leq 4$, find the conditional density $f_{X|Y}(x|y)$ of X, given $Y = y$.

   (c) Find the conditional probability that $X \leq 1$, given $1 \leq Y \leq 3$.

5. Let $X_1, X_2, ..., X_n$ be independent exponential random variables, with means $1/\lambda_1, 1/\lambda_2, ..., 1/\lambda_n$, respectively. Find the probability density function of $Y = \min(X_1, X_2, ..., X_n)$. *You must show all work to receive full credit.*