Lesson 17: Wrap-up and other regressions

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Animals of the day




Today

- Let’s zoom out a little and see what types of regressions we can do
- You should have the main tools to perform these regressions
  - Each has some nuances, but I’ll give you sources that help walk you through them

Types of regressions

Dist'n of Y	Typical uses	Link name	Link function	Common name
Normal 5/2	Linear-response data	Identity	$g(\mu) = \mu$	Linear regression
Bernoulli / Binomial 5/3	outcome of single yes/no occurrence	Logit	$g(\mu) = \text{logit}(\mu)$	Logistic regression
Poisson, 5/3	count of occurrences in fixed amount of time/space	Log	$g(\mu) = \log(\mu)$	Poisson regression
Bernoulli / Binomial	outcome of single yes/no occurrence	Log	$g(\mu) = \log(\mu)$	Log-binomial regression
Multinomial	outcome of single occurence with K > 2 options, *nominal*	Logit	$g(\mu) = \text{logit}(\mu)$	Multinomial logistic regression
Multinomial	outcome of single occurence with K > 2 options, *ordinal*	Logit	$g(\mu) = \text{logit}(\mu)$	Ordinal logistic regression

Linear regression

- **Outcome type:** continuous

- **Example outcomes:**
  - Height
  - IAT score
  - Heart rate

- **Population model**

$$E(Y \mid X) = \mu = \beta_0 + \beta_1 X$$

SYS

- **Interpretations**
  - The change in average Y for every 1 unit increase in X

Linear regression resources

- 512/612 class site!!
- Online textbook by Dr. Nahhas

Logistic regression

- **Outcome type:** binary, yes or no
- **Example outcomes:**
  - Food insecurity
  - Disease diagnosis for patient
  - Fracture
- **Population model**

$$\text{logit}(\mu) = \beta_0 + \beta_1 X$$

- **Interpretations**
  - The log-odds ratio for every 1 unit increase in X

Logistic regression resources

- Online textbook by Dr. Nahhas

Poisson Regression

- **Outcome type:** Counts or rates

- **Example outcomes:**
  - Number of children in household
  - Number of hospital admissions
  - Rate of incidence for COVID in US counties

- **Population model**

$$\log(\mu) = \beta_0 + \beta_1 X$$

- **Interpretations**
  - The count (or rate) ratio for every 1 unit increase in X

$\exp(\beta_1)$

β_1 : log count ratio

Poisson Regression resources

- PennState 504 website
- Online textbook by Dr. Nahhas
- YouTube video on R tutorial for Poisson Regression
  - Dr. Fogerty is a professor in Political Science, so just beware they may not have formal statistical training
- Guided R tutorial page on Poisson regression
- Online textbook by Dr. Werth
  - Social scientist, so just beware they may not have formal statistical training

Log-binomial Regression

- Outcome type: binary, yes or no
- Example outcomes:
  - Food insecurity
  - Disease diagnosis for patient
  - Fracture
- Population model

$$\log(\mu) = \beta_0 + \beta_1 X$$

$[0, \infty) \leftrightarrow (-\infty, \infty)$

- Interpretations
  - We have log of probability on the left
  - So exponential of our coefficients will be **risk ratio**

$$\beta_1 = \log(P(Y|X=x+1)) - \log(P(X=x))$$

$$\beta_1 = \log\left(\frac{P(Y|X=x+1)}{P(Y|X=x)}\right)$$

log risk ratio

Log-binomial Regression resources

- Online textbook by Dr. Nahhas
- Article on `logbin` package that is used to fit log-binomial regression

Multinomial logistic regression

- **Outcome type:** multi-level categorical, no inherent order
- **Example outcomes:**
  - Blood type
  - US region (from WBNS)
  - Primary site of lung cancer (upper lobe, lower lobe, overlapped, etc.)
- We have additional restriction that the multiple group probabilities sum to 1

$p(Y=0) + p(Y=1) + p(Y=2) = 1$

$\text{logit}(\mu_{gp2}) \rightarrow \log\left(\frac{\mu_{gp2}}{1-\mu_{gp2}}\right)$

3 gps

- **Population models**

$$\log\left(\frac{\mu_{\text{group 2}}}{\mu_{\text{group 1}}}\right) = \beta_0 + \beta_1 X$$

$$\log\left(\frac{\mu_{\text{group 3}}}{\mu_{\text{group 1}}}\right) = \beta_0 + \beta_1 X$$

gp 3 vs gp 2

- **Interpretations**
  - Basically fitting two binary logistic regressions at same time!
  - First equation: how a one unit change in X changes the log-odds of going from group 1 to group 2
  - Second equation: how a one unit change in X changes the log-odds of going from group 1 to group 3

Multinomial logistic regression resources

- YouTube video on R tutorial for Poisson Regression
  - Again, Dr. Fogerty is a professor in Political Science
- R-bloggers post with guided R code
- Online textbook by Dr. Werth with data and R script

Ordinal logistic regression

- **Outcome type:** multi-level categorical, with inherent order
- **Example outcomes:**
  - Satisfaction level (likert scale)
  - Pain level
  - Stages of cancer
- When these variables are predictors, we are pretty lenient about treating them as continuous
  - We must be VERY STRICT when they are outcomes
  - They do not meet the assumptions we place on continuous outcomes in linear regression!!
- We have additional restriction that the multiple group probabilities sum to 1

- **Population models** , with levels $k = 1, 2, 3, \ldots, K$

$$\log\left(\frac{P(Y \leq 1)}{P(Y > 1)}\right) = \beta_0 + \beta_1 X$$

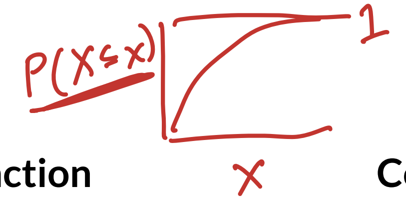
$$\log\left(\frac{P(Y \leq k)}{P(Y > k)}\right) = \beta_0 + \beta_1 X$$

- **Interpretations**
  - Basically fitting K binary logistic regressions at same time!
  - First equation: how a one unit change in X changes the log-odds of going from group 1 to any other group
  - Second equation: how a one unit change in X changes the log-odds of going from group 1 or 2 to group 3 or above

Ordinal logistic regression resources

- Online textbook by Dr. Nahhas
- Online textbook by Dr. Werth with data and R script

Even more regressions...



Dist'n of Y	Typical uses	Link name	Link function	Common name
Bernoulli / Binomial	outcome of single yes/no occurrence	Probit	$g(\mu) = \Phi^{-1}(\mu)$	Probit regression
Bernoulli / Binomial	outcome of single yes/no occurrence	Complementary log-log	$g(\mu) = \log(-\log(1-\mu))$	Complementary log-log regression
Multinomial	outcome of single occurence with K > 2 options, *nominal*	Probit	$g(\mu) = \Phi^{-1}(\mu)$	Multinomial probit regression
Multinomial	outcome of single occurence with K > 2 options, *ordinal*	Probit	$g(\mu) = \Phi^{-1}(\mu)$	Ordered probit regression

inverse CDF

More regression resources

- Probit regression
- Complementary log-log
- Multinomial probit
- Ordered probit

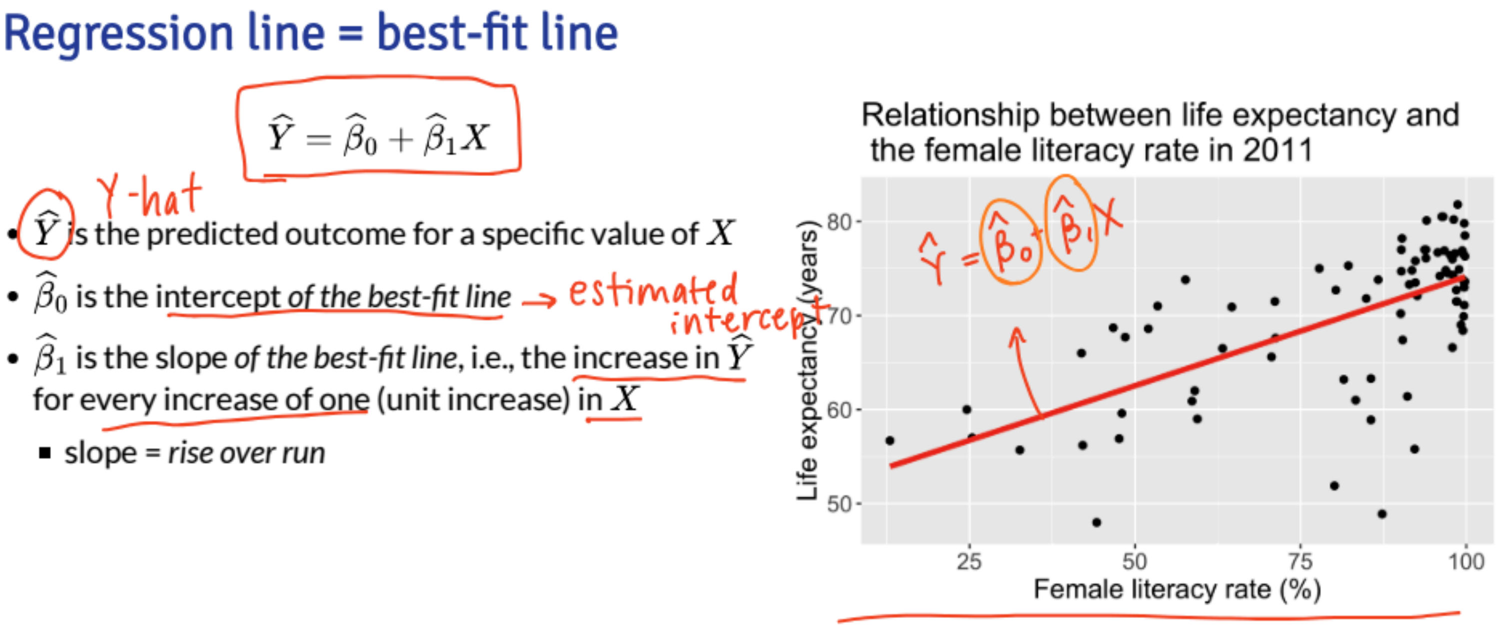
General resources

- Dr. Fogerty's YouTube series
- Dr. Werth's Categorical Book
- Dr. Nahhas' Book
- The Epidemiologist R Handbook
  - Analysis AND R work

Biostatisticians

Moment of appreciation for your growth

- Remember when we were learning simple linear regression…
- This was a slide from our second week together:



- Even if you don't feel like you learned everything, you have learned a lot from the first time you saw the above slide