MLR: Inference

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Learning Objectives

1. Interpret MLR (population) coefficient estimates with additional variable in model
2. Understand the use of the general F-test and interpret what it measures.
3. Understand the context of the **Overall F-test**, conduct the needed hypothesis test, and interpret the results.
4. Understand the context of the **single covariate F-test**, conduct the needed hypothesis test, and interpret the results.
5. Understand the context of the **group of covariates F-test**, conduct the needed hypothesis test, and interpret the results.

Let's map that to our regression analysis process

Model Selection

- Building a model
- Selecting variables
- Prediction vs interpretation
- Comparing potential models

Model Fitting

- Find best fit line
- Using OLS in this class
- Parameter estimation
- Categorical covariates
- Interactions

Model Evaluation

- Evaluation of model fit
- Testing model assumptions
- Residuals
- Transformations
- Influential points
- Multicollinearity

Model Use (Inference)

- Inference for coefficients
- Hypothesis testing for coefficients
- Inference for expected Y given X

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Interpreting the estimated population coefficients

- For a population model:

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \epsilon$$

  - Where X_1 and X_2 are continuous variables
  - No need to specify Y because it required to be continuous in linear regression

General interpretation for $\widehat{\beta}_0$

The expected Y-variable is ($\widehat{\beta}_0$ units) when the X_1-variable is 0 X_1-units and X_2-variable is 0 X_1 -units (95% CI: LB, UB).

General interpretation for $\widehat{\beta}_1$

For every increase of 1 X_1-unit in the X_1-variable, adjusting/controlling for X_2-variable, there is an expected increase/decrease of $|\widehat{\beta}_1|$ units in the Y-variable (95%: LB, UB).

General interpretation for $\widehat{\beta}_2$

For every increase of 1 X_2-unit in the X_2-variable, adjusting/controlling for X_1-variable, there is an expected increase/decrease of $|\widehat{\beta}_2|$ units in the Y-variable (95%: LB, UB).

Getting these interpretations from our regression table

We fit the regression model in R and printed the regression table:

```
mr1 <- lm(LifeExpectancyYrs ~ FemaleLiteracyRate + FoodSupplykcPPD,
          data = gapm_sub)
```

term	estimate	std.error	statistic	p.value	conf.low	conf.high
(Intercept)	33.595	4.472	7.512	0.000	24.674	42.517
FemaleLiteracyRate	0.157	0.032	4.873	0.000	0.093	0.221
FoodSupplykcPPD	0.008	0.002	4.726	0.000	0.005	0.012

Fitted multiple regression model: $\widehat{\text{LE}} = 33.595 + 0.157\ \text{FLR} + 0.008\ \text{FS}$

Interpretation for $\widehat{\beta}_0$

The expected life expectancy is 33.595 years when the female literacy rate is 0% and food supply is 0 0 kcal PPD (95% CI: 24.674, 41.517).

Interpretation for $\widehat{\beta}_1$

For every 1% increase in the female literacy rate, adjusting for food supply, there is an expected increase of 0.157 years in the life expectancy (95%: 0.093, 0.221).

Interpretation for $\widehat{\beta}_2$

For every 1 kcal PPD increase in the food supply, adjusting for female literacy rate, there is an expected increase of 0.008 years in life expectancy (95%: 0.005, 0.012).

Let's just examine the general interpretation vs. the example

General interpretation for $\widehat{\beta}_0$

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General interpretation for $\widehat{\beta}_2$

For every increase of 1 X_2-unit in the X_2-variable, adjusting/controlling for X_1-variable, there is an expected increase/decrease of $|\widehat{\beta}_2|$ units in the Y-variable (95%: LB, UB).

Interpretation for $\widehat{\beta}_0$

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What we need in our interpretations of coefficients (reference)

- Units of Y
- Units of X
- Discussing intercept: Mean or average or expected before Y
- Discussing coefficient for continuous covariate: Mean or average or expected before difference, increase, or decrease
  - OR: Mean or average or expected before Y
  - Only need before difference or Y!!
- Confidence interval

- If other covariates in the model
  - Discussing intercept: Must state that variables are equal to 0
    - or at their centered value if centered!
  - Discussing coefficient for covariate: Must state "adjusting for all other variables", "Controlling for all other variables", or "Holding all other variables constant"
    - If only one other variable in the model, then replace "all other variables" with the single variable name

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5. Understand the context of the **group of covariates F-test**, conduct the needed hypothesis test, and interpret the results.

We must revisit our dear friend, the F-test!

https://www.writerswrite.co.za/foreshadowing/

Remember from Lesson 5: F-test vs. t-test for the population slope

The square of a t-distribution with $df = \nu$ is an F-distribution with $df = 1, \nu$

$$T_\nu^2 \sim F_{1,\nu}$$

- We can use **either F-test** or **t-test** to run the following hypothesis test:

$$\begin{aligned} H_0 &: \beta_1 = 0 \\ \text{vs. } H_A &: \beta_1 \neq 0 \end{aligned}$$

- Note that the F-test does not support one-sided alternative tests, but the t-test does!

Remember from Lesson 5: Planting a seed about the F-test

We can think about the hypothesis test for the slope...

Null H_0	Alternative H_1
$\beta_1 = 0$	$\beta_1 \neq 0$

in a slightly different way...

in SLR

Null model ($\beta_1 = 0$)	Alternative model ($\beta_1 \neq 0$)
• $Y = \beta_0 + \epsilon$ • Smaller (reduced) model	• $Y = \beta_0 + \beta_1 X + \epsilon$ • Larger (full) model

- In multiple linear regression, we can start using this framework to test multiple coefficient parameters at once
  - Decide whether or not to reject the smaller reduced model in favor of the larger full model
  - Cannot do this with the t-test!

We can extend this!!

We can create a hypothesis test for more than one coefficient at a time…

Null H_0	Alternative H_1
$\beta_1 = \beta_2 = 0$	$\beta_1 \neq 0$ and/or $\beta_2 \neq 0$

in a slightly different way…

Null model	Alternative* model
• $Y = \beta_0 + \epsilon$ • Smaller (reduced) model	• $Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \epsilon$ • Larger (full) model

*This is **not quite** the alternative, but if we reject the null, then this is the model we move forward with

Poll Everywhere Question 1

Join by Web **PollEv.com/nickywakim275**

Which of the following null hypotheses can we NOT test with the F-test? Use the following model: $Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \beta_4 X_4$

$\beta_1 = \beta_2 = \beta_3 = \beta_4 = 0$ — **5%**

$\beta_3 = \beta_4 = 0$ — **0%**

We can test all of these! — **80%**

Building a very important toolkit: three types of tests

Overall test

Does at least one of the covariates/predictors contribute significantly to the prediction of Y?

Test for addition of a single variable (covariate subset test)

Does the addition of one particular covariate add significantly to the prediction of Y achieved by other covariates already present in the model?

Test for addition of group of variables (covariate subset test)

Does the addition of some group of covariates add significantly to the prediction of Y achieved by other covariates already present in the model?

$$Y = \beta_0 + \boxed{\beta_1 X_1} + \varepsilon$$

$$Y = \beta_0 + \boxed{\beta_1 X_1} + \underline{\beta_2 X_2 + \beta_3 X_3} + \varepsilon$$

Variation: Explained vs. Unexplained

obs → mean of Y → fitted Y (Y given X defined by model)

$$\sum_{i=1}^{n}(Y_i - \overline{Y})^2 = \sum_{i=1}^{n}(\widehat{Y}_i - \overline{Y})^2 + \sum_{i=1}^{n}(Y_i - \widehat{Y}_i)^2$$

$$SSY = SSR + SSE$$

- $Y_i - \overline{Y}$ = the deviation of Y_i around the mean $\overline{Y}$
  - (the **total** amount deviation unexplained at $X_{i1}, \ldots, X_{ik}$).
- $\widehat{Y}_i - \overline{Y}$ = the deviation of the fitted value $\widehat{Y}_i$ around the mean $\overline{Y}$
  - (the amount deviation **explained** by the regression at $X_{i1}, \ldots, X_{ik}$).
- $Y_i - \widehat{Y}_i$ = the deviation of the observation Y around the fitted regression line
  - (the amount deviation **unexplained** by the regression at $X_{i1}, \ldots, X_{ik}$)

Another way to think of SSY, SSR, and SSE

- Let's create a data frame of each component within the SS's
  - Difference in SSY: $Y_i - \overline{Y}$ $\overline{Y}$
  - Difference in SSR: $\widehat{Y}_i - \overline{Y}$
  - Difference in SSE: $Y_i - \widehat{Y}_i$
- Using our simple linear regression model as an example:

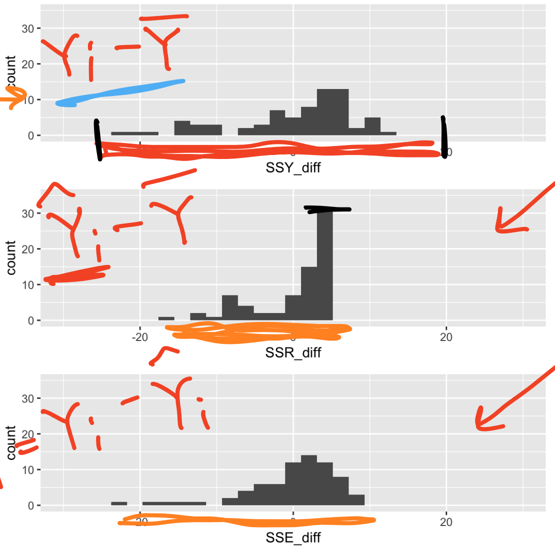
```
slr1 = lm(LifeExpectancyYrs ~ FemaleLiteracyRate, data = gapm_sub)
aug_slr1 = augment(slr1)
SS_df = gapm_sub %>% select(LifeExpectancyYrs) %>%
  mutate(SSY_diff = LifeExpectancyYrs - mean(LifeExpectancyYrs),
         y_fit = aug_slr1$.fitted,
         SSR_diff = y_fit - mean(LifeExpectancyYrs),
         SSE_diff = aug_slr1$.resid)
```

→ get $\hat{Y}_i$'s

→ give $Y_i - \hat{Y}_i$

Plot the components of each sum of squares

```
SSY_plot = ggplot(SS_df, aes(SSY_diff)) + geom_histogram() + xlim(-30, 30) + ylim(0
SSR_plot = ggplot(SS_df, aes(SSR_diff)) + geom_histogram() + xlim(-30, 30) + ylim(0
SSE_plot = ggplot(SS_df, aes(SSE_diff)) + geom_histogram() + xlim(-30, 30) + ylim(0
grid.arrange(SSY_plot, SSR_plot, SSE_plot, nrow = 3)
```



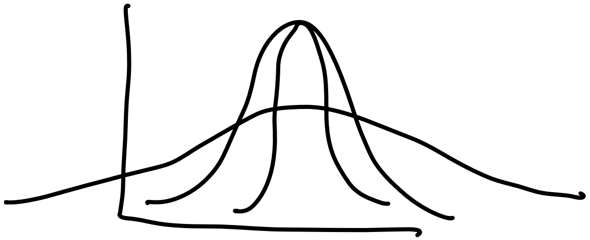
how much variation does the model explain

how much variation is left (unexplained) after model fit?

$$SSY = \sum_{i=1}^{n}(Y_i - \overline{Y})^2 = 64.64$$

$$SSR = \sum_{i=1}^{n}(\widehat{Y}_i - \overline{Y})^2 = 27.24$$

$$SSE = \sum_{i=1}^{n}(Y_i - \widehat{Y}_i)^2 = 37.39$$



When running a F-test for linear models...

- We need to define a larger, full model (more parameters)
- We need to define a smaller, reduced model (fewer parameters)
- Use the F-statistic to decide whether or not we reject the smaller model
  - The F-statistic compares the SSE of each model to determine if the full model explains a significant amount of additional variance

$$F = \frac{\frac{SSE(R) - SSE(F)}{df_R - df_F}}{\frac{SSE(F)}{df_F}}$$

reduced

- $SSE(R) \geq SSE(F)$

full

- Numerator measures difference in **unexplained** variation between the models
  - Big difference = added parameters greatly reduce the unexplained variation (increase explained variation)
  - Smaller difference = added parameters don't reduce the unexplained variation
- Take ratio of difference to the unexplained variation in the full model

Poll Everywhere Question 2

We will keep working with the MLR model from last class

New population model for example:

$$\text{Life expectancy} = \beta_0 + \beta_1 \text{Female literacy rate} + \beta_2 \text{Food supply} + \epsilon$$

```
# Fit regression model:
mr1 <- lm(LifeExpectancyYrs ~ FemaleLiteracyRate + FoodSupplykcPPD,
          data = gapm_sub)
tidy(mr1, conf.int=T) %>% gt() %>% tab_options(table.font.size = 35) %>% fmt_number
```

term	estimate	std.error	statistic	p.value	conf.low	conf.high
(Intercept)	33.595	4.472	7.512	0.000	24.674	42.517
FemaleLiteracyRate	0.157	0.032	4.873	0.000	0.093	0.221
FoodSupplykcPPD	0.008	0.002	4.726	0.000	0.005	0.012

Fitted multiple regression model:

$$\text{Life } \widehat{\text{expectancy}} = \widehat{\beta}_0 + \widehat{\beta}_1 \text{Female literacy rate} + \widehat{\beta}_2 \text{Food supply}$$

$$\text{Life } \widehat{\text{expectancy}} = 33.595 + 0.157 \text{ Female literacy rate} + 0.008 \text{ Food supply}$$

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Overall F-test

Does at least one of the covariates/predictors contribute significantly to the prediction of Y?

- For a general population MLR model,

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \ldots + \beta_k X_k + \epsilon$$

k covariates or predictors

We can create a hypothesis test for all the covariate coefficients…

Null H_0	Alternative H_1
$\beta_1 = \beta_2 = \ldots = \beta_k = 0$	At least one $\beta_j \neq 0$ (for $j = 1, 2, \ldots, k$)

Null / Smaller / Reduced model	Alternative / Larger / Full model
$Y = \beta_0 + \epsilon$	$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \ldots + \beta_k X_k + \epsilon$

Overall F-test: general steps for hypothesis test

SSE(R) or SSE_{Red}
SSE of reduced model

1. Met underlying LINE assumptions

2. State the null hypothesis

$$H_0: \beta_1 = \beta_2 = \ldots = \beta_k = 0$$
$$\text{vs. } H_A: \text{At least one } \beta_j \neq 0, \text{for } j = 1, 2, \ldots, k$$

3. Specify the significance level.

Often we use $\alpha = 0.05$

4. Specify the test statistic and its distribution under the null

The test statistic is F, and follows an F-distribution with numerator $df = k$ and denominator $df = n - k - 1$. (n = # obversation, k = # covariates)

5. Compute the value of the test statistic

The calculated **test statistic** is

$$\boxed{F = \frac{\frac{SSE(R) - SSE(F)}{df_R - df_F}}{\frac{SSE(F)}{df_F}}} = \frac{MSR_{full}}{MSE_{full}}$$

explained variance

unexplained variance

6. Calculate the p-value

We are generally calculating: $P(F_{k,n-k-1} > F)$

7. Write conclusion for hypothesis test

- Reject if: $P(F_{k,n-k-1} > F) < \alpha$

We (reject/fail to reject) the null hypothesis at the $100\alpha\%$ significance level. There is (sufficient/insufficient) evidence that at least one predictor's coefficient is not 0 (p-value = $P(F_{1,n-2} > F)$).

Overall F-test: a word on the conclusion

- If H_0 is rejected, we conclude there is sufficient evidence that at least one predictor's coefficient is different from zero.
- Same as: at least one independent variable contributes significantly to the prediction of Y

- If H_0 is not rejected, we conclude there is insufficient evidence that at least one predictor's coefficient is different from zero.
- Same as: Not enough evidence that at least one independent variable contributes significantly to the prediction of Y

Let's think about our MLR example for life expectancy

Our proposed population model

$$\text{LE} = \beta_0 + \beta_1 \text{FLR} + \beta_2 \text{FS} + \epsilon$$

Fitted multiple regression model:

$$\widehat{\text{LE}} = \widehat{\beta}_0 + \widehat{\beta}_1 \text{FLR} + \widehat{\beta}_2 \text{FS}$$
$$\widehat{\text{LE}} = 33.595 + 0.157\ \text{FLR} + 0.008\ \text{FS}$$

Our main question for the Overall F-test: Is the regression model containing female literacy rate and food supply useful in estimating countries' life expectancy?

Null / Smaller / Reduced model	Alternative / Larger / Full model
$LE = \beta_0 + \epsilon$	$LE = \beta_0 + \beta_1 FLR + \beta_2 FS + \epsilon$

Comparing the SSY, SSR, and SSE for reduced and full model

```
mod_red1 = lm(LifeExpectancyYrs ~ 1, data = gapm_sub)
aug_red1  = augment(mod_red1)

mod_full1 = lm(LifeExpectancyYrs ~ FemaleLiteracyRate + FoodSupplykcPPD,
                  data = gapm_sub)
aug_full1  = augment(mod_full1)

SS_df2 = gapm_sub %>% select(LifeExpectancyYrs) %>%
  mutate(SSY_diff_r1 = LifeExpectancyYrs - mean(LifeExpectancyYrs),
         SSR_diff_r1 = aug_red1$.fitted - mean(LifeExpectancyYrs),
         SSE_diff_r1 = aug_red1$.resid,
```

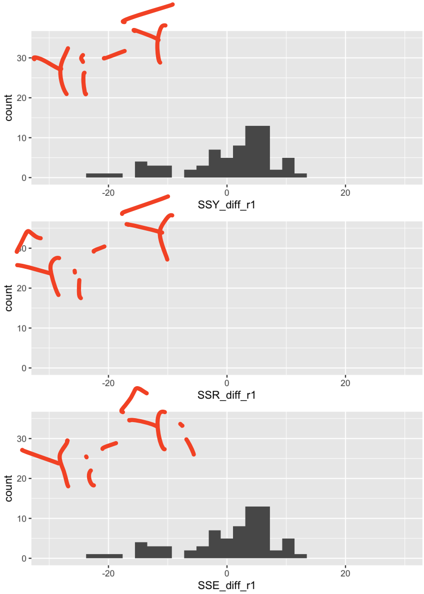
$Y \sim 1 \rightarrow Y = \beta_0 + \varepsilon$

$\rightarrow LE = \beta_0 + \beta_1 FLR + \beta_2 FS + \varepsilon$

Comparing the SSY, SSR, and SSE for reduced and full model

Reduced / null model

$$LE = \beta_0 + \epsilon$$



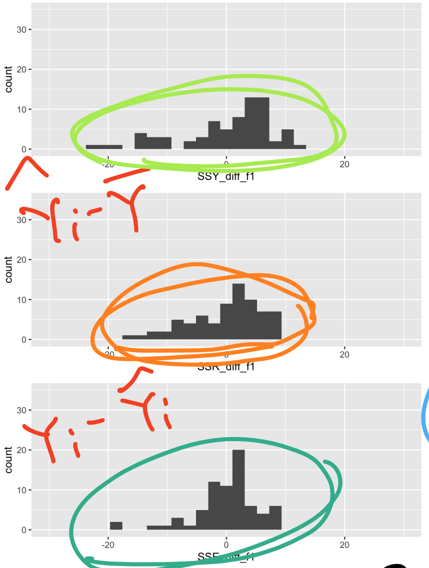
$SSY = 64.64$

$SSR = 0$

$SSE = 64.64$

Full / Alternative model

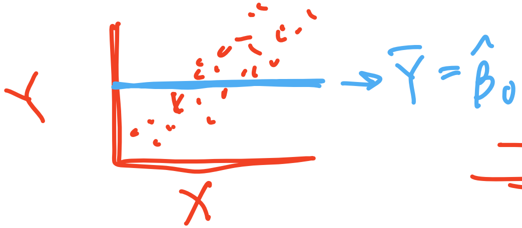
$$LE = \beta_0 + \beta_1 FLR + \beta_2 FS + \epsilon$$

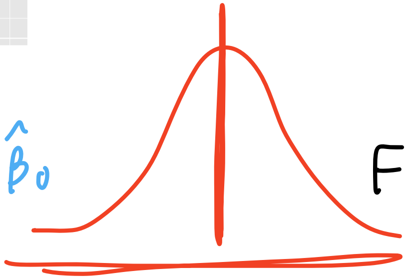


$SSY = 64.64$

$SSR = 36.39$

$SSE = 28.25$





$$F = \frac{\frac{SSE(R) - SSE(F)}{dfR - dfF}}{\frac{SSE(f)}{dfF}}$$

$$SSY = SSR + SSE$$

Poll Everywhere Question 3

Join by Web PollEv.com/nickywakim275

Using our SSE values of the full and reduced model, and the F-statistic equation, calculate the F-statistic. Note the df for the reduced model is 71 and the df for the full model is 69.

44.44

3 0

44.4

3 0

$$F = \frac{\frac{SSE(R) - SSE(F)}{dfR - dfF}}{\frac{SSE(f)}{dfF}}$$

$$= \frac{\left(\frac{64.64 - 28.25}{71-69}\right)}{\left(\frac{28.25}{69}\right)}$$

$$= 44.44$$

So let's step through our hypothesis test (1/3)

1. Met underlying LINE assumptions

2. State the null hypothesis

$$
\begin{aligned}
H_0 &: \beta_1 = \beta_2 = 0 \\
\text{vs. } H_A &: \text{At least one } \beta_1 \neq 0 \text{ or } \beta_2 \neq 0
\end{aligned}
$$

3. Specify the significance level

Often we use $\alpha = 0.05$

4. Specify the test statistic and its distribution under the null

The test statistic is F, and follows an F-distribution with numerator $df = k = 2$ and denominator $df = n - k - 1 = 72 - 2 - 1 = 69$. ($n$ = # obversation, k = # covariates)

So let's step through our hypothesis test (2/3)

5. Compute the value of the test statistic / 6. Calculate the p-value

4/5/6

The calculated **test statistic** is

$$F = \frac{\frac{SSE(R) - SSE(F)}{df_R - df_F}}{\frac{SSE(F)}{df_F}} = 44.443$$

OR use ANOVA table:

```
anova(mod_red1, mod_full1) %>% tidy() %>% gt() %>% tab_options(table.font.size = 35
```

4/5/6

term	df.residual	rss	df	sumsq	statistic	p.value
LifeExpectancyYrs ~ 1	71.000	4,589.119	NA	NA	NA	NA
LifeExpectancyYrs ~ FemaleLiteracyRate + FoodSupplykcPPD	69.000	2,005.556	2.000	2,583.563	44.443	0.000

So let's step through our hypothesis test (3/3)

7. Write conclusion for hypothesis test

We reject the null hypothesis at the 5% significance level. There is sufficient evidence that either countries' female literacy rate or the food supply (or both) contributes significantly to the ~~prediction~~ of life expectancy (p-value < 0.001).

estimation

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Covariate subset test: Single variable

Does the addition of one particular covariate of interest add significantly to the prediction of Y achieved by other covariates already present in the model?

- For a general population MLR model,

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_j X_j + \ldots + \beta_k X_k + \epsilon$$

We can create a hypothesis test for a single j covariate coefficient (where j can be any value $1, 2, \ldots, k$)...

Null H_0	Alternative H_1
$\beta_j = 0$	$\beta_j \neq 0$

Null / Smaller / Reduced model	Alternative / Larger / Full model
$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \ldots + \beta_k X_k + \epsilon$	$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_j X_j + \ldots + \beta_k X_k + \epsilon$

testing X_2: $Y = \beta_0 + \beta_1 X_1 + \beta_3 X_3 + \ldots + \beta_k X_k + \varepsilon$

Single covariate F-test: general steps for hypothesis test (reference)

1. Met underlying LINE assumptions

2. State the null hypothesis

$$H_0 : \beta_j = 0$$
$$\text{vs. } H_A : \beta_j \neq 0$$

3. Specify the significance level

Often we use $\alpha = 0.05$

4. Specify the test statistic and its distribution under the null

The test statistic is F, and follows an F-distribution with numerator $df = k$ and denominator $df = n - k - 1$. (n = # obversation, k = # covariates) testing

5. Compute the value of the test statistic

The calculated **test statistic** is

$$F = \frac{\frac{SSE(R) - SSE(F)}{df_R - df_F}}{\frac{SSE(F)}{df_F}}$$

6. Calculate the p-value

We are generally calculating: $P(F_{k,n-k-1} > F)$

7. Write conclusion for hypothesis test

We (reject/fail to reject) the null hypothesis at the $100\alpha\%$ significance level. There is (sufficient/insufficient) evidence that predictor/covariate j significantly improves the prediction of Y, given all the other covariates are in the model (p-value = $P(F_{1,n-2} > F)$).

Let's think about our MLR example for life expectancy

Our proposed population model

$$\text{LE} = \beta_0 + \beta_1\text{FLR} + \beta_2\text{FS} + \epsilon$$

Fitted multiple regression model:

$$\widehat{\text{LE}} = \widehat{\beta}_0 + \widehat{\beta}_1\text{FLR} + \widehat{\beta}_2\text{FS}$$
$$\widehat{\text{LE}} = 33.595 + 0.157\ \text{FLR} + 0.008\ \text{FS}$$

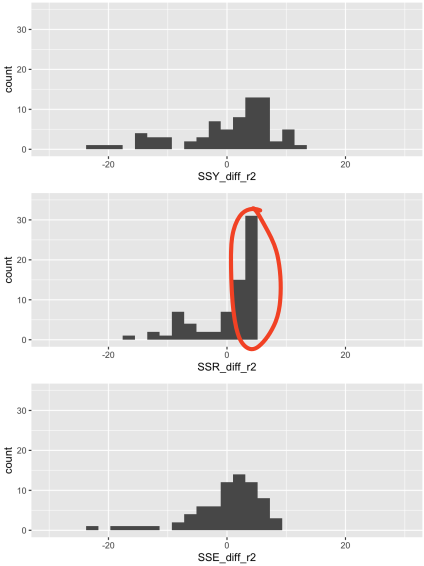
Our main question for the single covariate subset F-test: Is the regression model containing food supply improve the estimation of countries' life expectancy, given female literacy rate is already in the model?

Null / Smaller / Reduced model	Alternative / Larger / Full model
$LE = \beta_0 + \beta_1 FLR + \epsilon$	$LE = \beta_0 + \beta_1 FLR + \beta_2 FS + \epsilon$

Comparing the SSY, SSR, and SSE for reduced and full model

Reduced / null model

$$LE = \beta_0 + \beta_1 FLR + \epsilon$$



$SSY = 64.64$

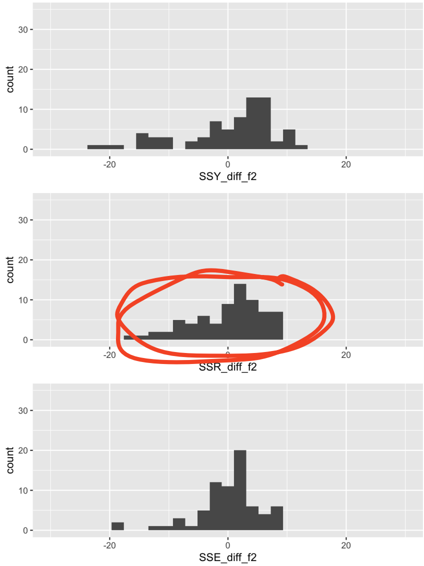
↳ SST

$SSR = 27.24$

$SSE = 37.39$

Full / Alternative model

$$LE = \beta_0 + \beta_1 FLR + \beta_2 FS + \epsilon$$



$SSY = 64.64$

$SSR = 36.39$

$SSE = 28.25$

$$SSE_{Red} \geq SSE_{full}$$

Poll Everywhere Question 4

Activity is now locked.
Responses are not accepted at this time.

For the reduced and full models below, what are possible SSE's for each model if SSY=64.64?
reduced: $LE = \beta_0 + \beta_1 FLR + \epsilon$

SSE(red) = 20 SSE(full) = 40

35%

SSE(red) = 70, SSE(full) = 20

10%

SSE(red) = 40, SSE(full) = 20

50%

$SSE_{red} \geq SSE_{full}$

$SSY = 64$

$\hookrightarrow SSY = SSE + SSR$

$SSE \leq SSY$

So let's step through our hypothesis test (1/3)

1. Met underlying LINE assumptions

2. State the null hypothesis

$$H_0 : \beta_2 = 0$$
$$\text{vs. } H_A : \beta_2 \neq 0$$

3. Specify the significance level

Often we use $\alpha = 0.05$

4. Specify the test statistic and its distribution under the null

The test statistic is F, and follows an F-distribution with numerator $df = k = 2$ and denominator $df = n - k - 1 = 72 - 2 - 1 = 69$. ($n$ = # obversation, k = # covariates)

So let's step through our hypothesis test (2/3)

5. Compute the value of the test statistic / 6. Calculate the p-value

The calculated **test statistic** is

$$F = \frac{\frac{SSE(R) - SSE(F)}{df_R - df_F}}{\frac{SSE(F)}{df_F}}$$

ANOVA table:

```
anova(mod_red2, mod_full2) %>% tidy() %>% gt() %>% tab_options(table.font.size = 35
```

term	df.residual	rss	df	sumsq	statistic	p.value
LifeExpectancyYrs ~ FemaleLiteracyRate	70.000	2,654.875	NA	NA	NA	NA
LifeExpectancyYrs ~ FemaleLiteracyRate + FoodSupplykcPPD	69.000	2,005.556	1.000	649.319	22.339	0.000

pval < 0.001

⇒ reject H_0

So let's step through our hypothesis test (3/3)

7. Write conclusion for hypothesis test

We reject the null hypothesis at the 5% significance level. There is sufficient evidence that countries' food supply contributes significantly to the prediction of life expectancy, given that female literacy rate is already in the model (p-value < 0.001).

Learning Objectives

1. Interpret MLR (population) coefficient estimates with additional variable in model
2. Understand the use of the general F-test and interpret what it measures.
3. Understand the context of the **Overall F-test**, conduct the needed hypothesis test, and interpret the results.
4. Understand the context of the **single covariate F-test**, conduct the needed hypothesis test, and interpret the results.
5. Understand the context of the **group of covariates F-test**, conduct the needed hypothesis test, and interpret the results.

Subset of covariates

Covariate subset test: group of variables

Does the addition of some group of covariates of interest add significantly to the prediction of Y obtained through other independent variables already present in the model?

- For a general population MLR model,

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \ldots + \beta_k X_k + \epsilon$$

for our ex on slide $k > 3$

We can create a hypothesis test for a group of covariate coefficients (subset of many)... **For example...**

Null H_0	Alternative H_1
$\beta_1 = \beta_3 = 0$ (this can be any coefficients)	At least one $\beta_j \neq 0$ (for $j = 2, 3$)

$\beta_2 = \beta_3 = 0$ or $\beta_1 = \beta_2 = 0$

Null / Smaller / Reduced model	Alternative / Larger / Full model
$Y = \beta_0 + \beta_2 X_2 + \epsilon$	$Y = \beta_0 + \beta_1 X + \beta_2 X + \beta_3 X_3 + \epsilon$

Covariate subset F-test: general steps for hypothesis test (reference)

1. Met underlying LINE assumptions

2. State the null hypothesis

For example:

$$H_0: \beta_1 = \beta_3 = 0$$
$$\text{vs. } H_A: \text{At least one } \beta_j \neq 0, \text{for } j = 1, 3$$

3. Specify the significance level

Often we use $\alpha = 0.05$

4. Specify the test statistic and its distribution under the null

The test statistic is F, and follows an F-distribution with numerator $df = k$ and denominator $df = n - k - 1$. (n = # obversation, k = # covariates)

5. Compute the value of the test statistic

The calculated **test statistic** is

$$F = \frac{\frac{SSE(R) - SSE(F)}{df_R - df_F}}{\frac{SSE(F)}{df_F}}$$

6. Calculate the p-value

We are generally calculating: $P(F_{k,n-k-1} > F)$

7. Write conclusion for hypothesis test

We (reject/fail to reject) the null hypothesis at the $100\alpha\%$ significance level. There is (sufficient/insufficient) evidence that predictors/covariates $2, 3$ significantly improve the prediction of Y, given all the other covariates are in the model (p-value = $P(F_{1,n-2} > F)$).

We need to slightly alter our MLR example for life expectancy

Our proposed population model to include water source percent (WS):

$$\text{LE} = \beta_0 + \beta_1 \text{FLR} + \beta_2 \text{FS} + \beta_3 WS + \epsilon$$

- We don't have a fitted multiple regression model for this yet!

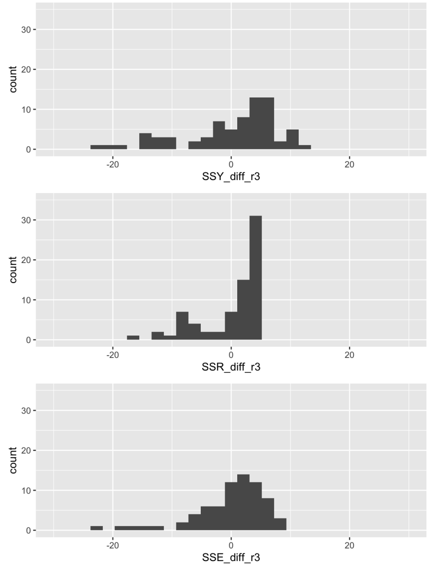
Our main question for the group covariate subset F-test: Is the regression model containing food supply and water source percent improve the estimation of countries' life expectancy, given percent female literacy rate is already in the model?

Null / Smaller / Reduced model	Alternative / Larger / Full model
$LE = \beta_0 + \beta_1 FLR + \epsilon$	$LE = \beta_0 + \beta_1 FLR + \beta_2 FS + \beta_3 WS + \epsilon$

Comparing the SSY, SSR, and SSE for reduced and full model

Reduced / null model

$$LE = \beta_0 + \beta_1 FLR + \epsilon$$



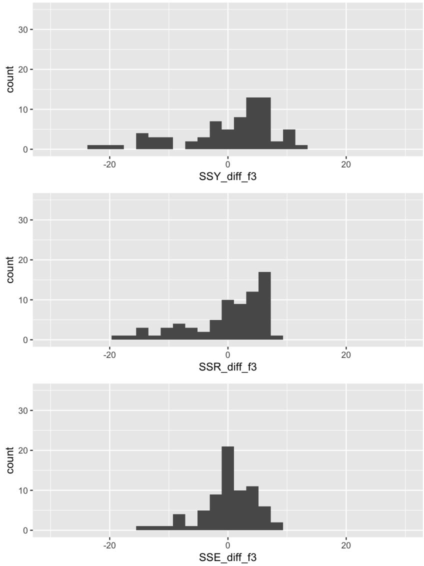
$$SSY = 64.64$$

$$SSR = 27.24$$

$$SSE = 37.39$$

Full / Alternative model

$$LE = \beta_0 + \beta_1 FLR + \beta_2 FS + \beta_3 WS + \epsilon$$



$$SSY = 64.64$$

$$SSR = 43.26$$

$$SSE = 21.38$$

So let's step through our hypothesis test (1/3)

1. Met underlying LINE assumptions

2. State the null hypothesis

$$\begin{aligned} H_0 &: \beta_2 = \beta_3 = 0 \\ \text{vs. } H_A &: \beta_2 \neq 0 \text{ and/or } \beta_3 \neq 0 \end{aligned}$$

3. Specify the significance level

Often we use $\alpha = 0.05$

4. Specify the test statistic and its distribution under the null

The test statistic is F, and follows an F-distribution with numerator $df = k = 2$ and denominator $df = n - k - 1 = 72 - 2 - 1 = 69$. ($n$ = # obversation, k = # covariates)

So let's step through our hypothesis test (2/3)

5. Compute the value of the test statistic / 6. Calculate the p-value

The calculated **test statistic** is

$$F = \frac{\frac{SSE(R) - SSE(F)}{df_R - df_F}}{\frac{SSE(F)}{df_F}}$$

ANOVA table:

```
anova(mod_red3, mod_full3) %>% tidy() %>% gt() %>% tab_options(table.font.size = 35
```

term	df.residual	rss	df	sumsq	statistic	p.value
LifeExpectancyYrs ~ FemaleLiteracyRate	70.000	2,654.875	NA	NA	NA	NA
LifeExpectancyYrs ~ FemaleLiteracyRate + FoodSupplykcPPD + WaterSourcePrct	68.000	1,517.916	2.000	1,136.959	25.467	0.000

So let's step through our hypothesis test (3/3)

7. Write conclusion for hypothesis test

We reject the null hypothesis at the 5% significance level. There is sufficient evidence that countries' food supply or water source (or both) contribute significantly to the prediction of life expectancy, given that female literacy rate is already in the model (p-value < 0.001).

Other ways to word the hypothesis tests (reference)

- Single covariate subset F-test $H_0: \beta_1 = 0 \quad H_A: \beta_1 \neq 0$
  - H_0 : X^* does not significantly improve the prediction of Y, given that $X_1, X_2, \ldots, X_p$ are already in the model X_1
  - H_A : X^* significantly improves the prediction of Y, given that $X_1, X_2, \ldots, X_p$ are already in the model
- Group covariate subset F-test
  - H_0 : The addition of the s variables $X_1^*, X_2^*, \ldots, X_s^*$ does not significantly improve the prediction of Y, given that $X_1, X_2, \ldots, X_q$ are already in the model
  - H_A : The addition of the s variables $X_1^*, X_2^*, \ldots, X_s^*$ significantly improves the prediction of Y, given that $X_1, X_2, \ldots, X_q$ are already in the model

SSY SSR SSE relationship SLR

↳ explained w/ scatterplot AND histogram

$Y = \beta_0 + \varepsilon$

$Y = \beta_0 + \beta_1 X + \varepsilon$